The (virtual) conceptual necessity of quantum probabilities in cognitive psychology

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Abstract: We propose a way in which Pothos and Busemeyer could strengthen their position. Taking a dynamic stance we consider cognitive tests as functions that transfer a given input state into the state after testing. Under very general conditions it can be shown that testable properties in cognition form an orthomodular lattice. Gleason's theorem then yields the conceptual necessity of QP.

Pothos and Busemeyer (P&B) discuss quantum probabilities (QP) as providing an alternative to classical probability (CP) for understanding cognition. In considerable detail, they point out several phenomena CP cannot explain and they demonstrate how QP can account for these phenomena. An obvious way to downplay this chain of arguments is by demonstrating that besides CP and QP models, alternative approaches are possible that could also describe the phenomena without using the strange and demanding instrument of QP. For instance, one could argue that the conjunction puzzle can be resolved by simple heuristics (Gigerenzer, 1997), and the question ordering effects by query theory (Johnson, Haubl, & Keinan, 2007).

A general strategy to invalidate such criticism is to look for a universal motivation of QP which is based on fundamental (architectural) properties of the area under investigation. As Kuhn (1996) clarified, such basic assumptions constituting a theoretical paradigm normally cannot be justified empirically. Basic assumptions which concern the general architecture of the theoretical system (paradigm) are called *design features*. Using a term that is common in the generative linguistic literature (Chomsky, 1995, 2005), we shall call properties that are consequences of such design features as applying with (*virtual*) *conceptual necessity*.

We believe that P&B could further strengthen their argument by demonstrating that quantum probabilities are such a (virtual) conceptual necessity. This can be achieved by adopting the recent developments of "dynamification" in logic (van Benthem, 2011) and cognition (Barsalou, 2008) where cognitive actions play a much more significant role than "static" propositions. Among others (including Atmanspacher, Römer, & Walach, 2002), Baltag & Smets (2005) have given a complete axiomatization for quantum action, based on the idea of a quantum transition systems. In this view, the states of a system are identified with the actions that can be performed on the states. In physics, the relevant actions are measurements. In cognitive psychology, actions correspond to tests that subjects carry out (yes/no questions in the simplest case). Basically, a *quantum dynamic frame* is characterized by a set of states Σ and a system T of subsets of Σ called testable properties. Each testable property $A \in \mathrm{T}$ is characterized by a unique (partial) transfer function P_A which describes how testing of *A* changes an input state $s \in \Sigma$. The system of testable properties and the corresponding transfer functions are characterized by some plausibility conditions. For instance, the testable properties are closed under arbitrary intersection, states are testable, testing a true property does not change the state ($s \in A$ implies $P_A(s)=s$), repeatability ($P_A^2= P_A$). Further, there are more technical axioms such as self-adjointness and a covering law.

Restricting testable properties in this way, Baltag & Smets (2005) were able to prove (based on earlier work by Piron 1976) that quantum dynamic frames are isomorphic to the lattice of the closed subspaces of a Hilbert space (with transfer functions as projection operators). In the Baltag/Smets approach, two states *s* and *t* are considered orthogonal if no measurement can transfer *s* into *t*. Properties *A* and *B* are orthogonal if all states of *A* ore orthogonal to all states of *B*. If *A* and *B* are orthogonal, the corresponding subspaces of the Hilbert space are orthogonal as well (and *vice versa*). Mathematically, probabilities are *totally additive measure functions* – in the classical case based on Boolean algebras and in the quantum case based on orthomodular lattices. The underlying algebra is decisive for the properties of the resulting measure function. In the quantum case, Gleason's theorem states that the corresponding measure functions can be expressed by the squared length of the projections of a given state *s* (or more generally, as the convex hull of such functions; for details, see the original paper Gleason (1957), and for a constructive proof see Richman & Bridges (1999)), i.e., as QP.

Our view is further supported by P&B's speculations about implications for brain neurophysiology. In the algebraic approach, even classical dynamical systems such as neural networks, could exhibit quantum-like properties in the case of coarse-graining measurements, when testing a property cannot distinguish between epistemically equivalent states. Beim Graben & Atmanspacher (2009) used this "epistemic quantization" for proving the possibility of incompatible properties in classical dynamical systems. In neuroscience, most measurements, such as electroencephalography or magnetic resonance imaging, are coarse-grainings in this sense. Thus, the Baltag/Smets approach has direct implications for brain neurophysiology, without needing to refer to a "quantum brain" as indicated by P&B.

Taken together, the Baltag/Smets approach provides an independent motivation of QP which is not based on particular phenomena but rather on independently motivated general conditions concerning the dynamics of testing. All the conditions needed for the proof are formulated in purely dynamic terms. This makes quantum dynamic frames especially appealing for psychological approaches formulating operational cognitive laws. Recent work by Busemeyer & Bruza (2012), Trueblood & Busemeyer (2011), and Blutner (2012) is in this spirit. Since based on a dynamic picture of propositions and questions and hence on the design principles of cognitive architecture, we state that QP are a virtual conceptual necessity. Needless to say, that we regain CP (Kolmogorovian) by assuming that no test is changing the state being tested.

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