Complementarity in cognition entailed by bounded rationality

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Recent research has revealed intriguing parallels between quantum physics and cognitive sciences dealing with concepts, vagueness, and prototypicality[1]. These parallels become obvious when one of the key features of quantum theory is considered: the concept of *complementarity*.

Two observables are called complementary when a pure state cannot be a common eigenstate of both observables, which is not possible in classical physics where pure states are identified with points in a phase space that are common eigenstates of every observable. The situation is different, however, if one also takes statistical states, namely dispersive probability measures upon phase space, into account. Then, the concept of a quantum mechanical eigenstate applies straightforwardly to an operationally restricted state space resulting from a phase space *coarse-graining* into epistemic equivalence classes. This 'epistemic quantization' of classical dynamical systems introduced by beim Graben *et al.*[2] is nicely illustrated by Foulis' firefly box[3] where an observer is only able to tell the glowing firefly's position either as to the right or to the left with respect to the front window or to the front or to the bottom with respect to the side window.

Here, we suggest to regard this operational constraint, preventing the simultaneous assessment of two complementary perspectives, being caused by limited resources as in *bounded rationality*[4]. Considering the origin of complementary in cognition as a kind of bounded rationality leads to orthomodular lattices by pasting together two (or more) partial Boolean algebras as demonstrated with Foulis' firefly box[3,5].

However, Foulis' example only presents a static picture for the emergence of quantum-like descriptions from coarse-grained classical systems. If the firefly were exploring a chaotic itinerary, the dynamics would have to be taken into account as well. As beim Graben *et al.*[2] have demonstrated, a chaotic itinerary yields a *dynamic refinement* of the original coarse-graining that converges towards single points in phase space through the limit of continuous observations. But for general, arbitrary coarse-grainings the residual grains are not common eigenstates of any observable. Then, the Boolean partition algebras of the finest refinements of two (or more) arbitrary observables can again be pasted together along their overlaps into an orthomodular lattice[5] thus entailing the canonical Hilbert space representation exploited in quantum cognition[1].

References

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