Game Theoretic Pragmatics . . .

• aims for mathematically precise models of language use and interpretation . . .

• by formally representing interlocutors’:
  (i) action alternatives
  (ii) preferences
  (iii) beliefs about (i)–(iii).
Today’s Agenda

• introduce game theory
• charter how to apply GT to linguistic pragmatics
  ↦ IBR-model
• compare BiOT with IBR-model and reinforcement learning
  ↦ find an interpretation of strong/weak optimality
Running Example: Division of Pragmatic Labor

- Horn (1984)
- a.k.a. M-Implicature (Levinson, 2000)
- unmarked form pairs with unmarked meaning $m \leftrightarrow t$
- marked form pairs with marked meaning $m^* \leftrightarrow t^*$

Example 1 (Black Bart)

(1) a. Black Bart killed the sheriff.
   b. $\Rightarrow$ Black Bart killed the sheriff in a stereotypical way.

(2) a. Black Bart caused the sheriff to die.
   b. $\Rightarrow$ BB killed the sheriff in a non-stereotypical way.
Running Example: Division of Pragmatic Labor

- Horn (1984)
- a.k.a. M-Implicature (Levinson, 2000)
- unmarked form pairs with unmarked meaning \( m \leftrightarrow t \)
- marked form pairs with marked meaning \( m^* \leftrightarrow t^* \)

Example 1

(3) a. Black Bart killed the sheriff. \( m \)
   b. \( \sim \) Black Bart killed the sheriff in a stereotypical way. \( t \)

(4) a. Black Bart caused the sheriff to die. \( m^* \)
   b. \( \sim \) BB killed the sheriff in a non-stereotypical way. \( t^* \)
Example 2

(Sue’s Smile)

(5) a. Sue smiled.  
   b. $\leadsto$ Sue smiled genuinely.

(6) a. The corners of Sue’s lips turned slightly upwards.  
   b. $\leadsto$ Sue faked a smile.

Example 3

(Mrs T’s Song)

(7) a. Mrs T sang ‘Home Sweet Home.’  
   b. $\leadsto$ Mrs T sang a lovely song.

(8) a. Mrs T produced a series of sounds roughly corresponding to the score of ‘Home Sweet Home.’  
   b. $\leadsto$ Mrs T sang very badly.
Example 2

(Sue’s Smile)

(9)  a. Sue smiled.  
    b. $\rightsquigarrow$ Sue smiled genuinely.  

(10) a. The corners of Sue’s lips turned slightly upwards.  
    b. $\rightsquigarrow$ Sue faked a smile.

Example 3

(Mrs T’s Song)

(11) a. Mrs T sang ‘Home Sweet Home.’  
    b. $\rightsquigarrow$ Mrs T sang a lovely song.  

(12) a. Mrs T produced a series of sounds roughly corresponding to the score of ‘Home Sweet Home.’  
    b. $\rightsquigarrow$ Mrs T sang very badly.
biOT’s Explanation

\[ m \leftarrow m^* \]

\[ t \leftarrow t^* \]

generator & preferences

\[ m \quad \circ \quad \times \]

\[ m^* \quad \times \quad \bullet \]

\[ m \quad \circ \quad \times \]

\[ m^* \quad \times \quad \circ \]

strong optimality & blocking

strong optimality & weak optimality

explanation:

\bullet : possible
\circ : optimal
\times : blocked
**biOT’s Explanation**

- $m$ (generator & preferences)
- $m^*$
- $t$ ($t^*$): possible
- $\bigcirc$: optimal
- $\times$: blocked

**Explanation:**

- Strong optimality & blocking
- Strong optimality & weak optimality
**biOT’s Explanation**

- **Generator & preferences**
  - $t \leftarrow t^*$
  - $m \leftarrow m^*$

- **Strong optimality & blocking**
  - $t \leftarrow t^*$
  - $m \leftarrow m^*$

**explanation:**
- ● : possible
- ○ : optimal
- × : blocked
Controversial Issue

how to interpret strong/weak optimality?

1. online reasoning? (e.g. Hendriks et al., 2010)
2. diachronic optimization? (e.g. Blutner and Zeevat, 2008)
Branches of Game Theory

- **classical game theory** (since 1940)
  - (mostly) assumes perfectly rational agents
  - central notion: Nash equilibrium

- **evolutionary game theory** (since 1970)
  - long-term optimization of boundedly-rational agents
  - central notions: evolutionary stability & replicator dynamics

- **behavioral game theory** (since 1990)
  - studies interactive decision making in the lab
  - seeks regularities in choices beyond perfect rationality
Let’s Play: $p$-Beauty Contest (with $p = 2/3$)

Everybody choose and write down a number from 0 to 100 (including each). We will sum and average all choices. The person(s) closest to $2/3$ of the average will win.
Kinds of Games

<table>
<thead>
<tr>
<th>uncertainty</th>
<th>choice points</th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td>strategic/static</td>
</tr>
<tr>
<td></td>
<td>simultaneous</td>
</tr>
<tr>
<td>yes</td>
<td>Bayesian</td>
</tr>
<tr>
<td></td>
<td>in sequence</td>
</tr>
<tr>
<td></td>
<td>dynamic/sequential</td>
</tr>
<tr>
<td></td>
<td>with complete info</td>
</tr>
<tr>
<td></td>
<td>dynamic/sequential</td>
</tr>
<tr>
<td></td>
<td>with incomplete info</td>
</tr>
</tbody>
</table>
Games vs. Solutions

- **Game Models:**
  representations of a choice situation

- **Solutions Concepts:**
  capture *particular* behavior:
  good, optimal, rational, stable (…)

**Static Games**

- players choose simultaneously
- players have complete information

**Examples**

<table>
<thead>
<tr>
<th></th>
<th>$a_c$</th>
<th>$a_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_c$</td>
<td>2,2</td>
<td>0,3</td>
</tr>
<tr>
<td>$a_d$</td>
<td>3,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>

Prisoner’s Dilemma

<table>
<thead>
<tr>
<th></th>
<th>$a_{stay}$</th>
<th>$a_{go}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{stay}$</td>
<td>2,2</td>
<td>0,0</td>
</tr>
<tr>
<td>$a_{go}$</td>
<td>0,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>

Coordination Problem
Static Games

- players choose simultaneously
- players have complete information

Examples

<table>
<thead>
<tr>
<th></th>
<th>$a_c$</th>
<th>$a_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_c$</td>
<td>2,2</td>
<td>0,3</td>
</tr>
<tr>
<td>$a_d$</td>
<td>3,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>

Prisoner’s Dilemma

<table>
<thead>
<tr>
<th></th>
<th>$a_{stay}$</th>
<th>$a_{go}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{stay}$</td>
<td>2,2</td>
<td>0,0</td>
</tr>
<tr>
<td>$a_{go}$</td>
<td>0,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>

Coordination Problem
Static Games

- players choose simultaneously
- players have complete information

Examples

<table>
<thead>
<tr>
<th></th>
<th>$a_c$</th>
<th>$a_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_c$</td>
<td>2,2</td>
<td>0,3</td>
</tr>
<tr>
<td>$a_d$</td>
<td>3,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>

Prisoner’s Dilemma

<table>
<thead>
<tr>
<th></th>
<th>$a_{stay}$</th>
<th>$a_{go}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{stay}$</td>
<td>2,2</td>
<td>0,0</td>
</tr>
<tr>
<td>$a_{go}$</td>
<td>0,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>

Coordination Problem
Nash Equilibrium (Intuition)

Arrangement of strategies, one for each player, such that no player would benefit from unilateral deviation (i.e., no player would be better off doing something else if everybody else keeps doing the same thing).
Example (Prisoner’s Dilemma)

<table>
<thead>
<tr>
<th></th>
<th>$a_c$</th>
<th>$a_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_c$</td>
<td>2,2</td>
<td>0,3</td>
</tr>
<tr>
<td>$a_d$</td>
<td>3,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>

- single pure NE: $\langle a_d, a_d \rangle$
Example (Coordination)

<table>
<thead>
<tr>
<th></th>
<th>$a_{\text{stay}}$</th>
<th>$a_{\text{go}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{\text{stay}}$</td>
<td>2,2</td>
<td>0,0</td>
</tr>
<tr>
<td>$a_{\text{go}}$</td>
<td>0,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>

- two pure NEs: $\langle a_{\text{stay}}, a_{\text{stay}} \rangle$, and $\langle a_{\text{go}}, a_{\text{go}} \rangle$
Interpretation Game for DoPL

(Dekker and van Rooij, 2000)
Interpretation Game for DoPL

(Dekker and van Rooij, 2000)

<table>
<thead>
<tr>
<th></th>
<th>$t$</th>
<th>$t^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>$\bullet$ ← $\bullet$</td>
<td></td>
</tr>
<tr>
<td>$m^*$</td>
<td>$\bullet$ ← $\bullet$</td>
<td></td>
</tr>
</tbody>
</table>

BiOT-System

<table>
<thead>
<tr>
<th></th>
<th>$t$</th>
<th>$t^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>2,2</td>
<td>1,1</td>
</tr>
<tr>
<td>$m^*$</td>
<td>1,1</td>
<td>0,0</td>
</tr>
</tbody>
</table>

Static Game
Interpretation Game for DoPL

(Dekker and van Rooij, 2000)

<table>
<thead>
<tr>
<th></th>
<th>$t$</th>
<th>$t^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>2,2</td>
<td>1,1</td>
</tr>
<tr>
<td>$m^*$</td>
<td>1,1</td>
<td>0,0</td>
</tr>
</tbody>
</table>

Strong Optimality
Interpretation Game for DoPL

(Dekker and van Rooij, 2000)

\[
\begin{array}{ccc}
\text{ } & t & t' \\
\text{ } & \odot & \times \\
\text{ } & m' & \times & \bullet \\
\text{Strong Optimality} & & & \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{ } & t & t^* \\
\text{ } & \text{ } & \text{ } \\
m & 2,2 & 1,1 \\
m^* & 1,1 & 0,0 \\
\text{Nash Equilibrium} & & & \\
\end{array}
\]
Interpretation Game for DoPL

(Dekker and van Rooij, 2000)

\[
\begin{array}{c}
\begin{array}{c}
 t \\
m' \\
m
\end{array} & \begin{array}{c}
 t' \\
m' \\
m
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\begin{array}{cc}
 m & \bigcirc & \times \\
m' & \times & \bigcirc
\end{array}
\end{array}
\]

Weak Optimality

\[
\begin{array}{c}
\begin{array}{c}
 t \\
m
\end{array} & \begin{array}{c}
 t^* \\
m^*
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\begin{array}{c}
 m & 2,2 & 1,1 \\
m^* & 1,1 & 0,0
\end{array}
\end{array}
\]

Nash Equilibrium
Interpretation Game for DoPL

\[
\begin{array}{l}
\begin{array}{c}
 m \\
 m' \\
\end{array} \\
\begin{array}{c}
 ∘ \\
 × \\
\end{array} \\
\end{array}
\quad \begin{array}{c}
 t \\
 t' \\
\end{array}
\]

Weak Optimality

(Dekker and van Rooij, 2000)

\[
\begin{array}{l}
\begin{array}{c}
 m \\
 m' \\
 m^* \\
\end{array} \\
\begin{array}{c}
 2,2 \\
 1,1 \\
 0,0 \\
\end{array} \\
\end{array}
\quad \begin{array}{c}
 t \\
 t^* \\
\end{array}
\]

???
Static Games & BiOT (Dekker and van Rooij, 2000)

- BiOT-Systems $\leftrightarrow$ Static Games
- strong optimality $\leftrightarrow$ Nash equilibrium
- weak optimality $\leftrightarrow$ iterated Nash equilibrium (???)
Criticism

- static interpretation game means that:
  1. speaker and hearer choose simultaneously
  2. speaker chooses utterance independently of meaning that she wants to express
  3. hearer chooses interpretation independently of any utterance that needs to be interpreted
## Kinds of Games

<table>
<thead>
<tr>
<th>uncertainty</th>
<th>choice points</th>
</tr>
</thead>
<tbody>
<tr>
<td>simultaneous</td>
<td>in sequence</td>
</tr>
<tr>
<td>no</td>
<td>strategic/static</td>
</tr>
<tr>
<td></td>
<td>dynamic/sequential with complete info</td>
</tr>
<tr>
<td>yes</td>
<td>Bayesian</td>
</tr>
<tr>
<td></td>
<td>dynamic/sequential with incomplete info</td>
</tr>
<tr>
<td></td>
<td>U signaling games</td>
</tr>
<tr>
<td></td>
<td>U interpretation games</td>
</tr>
</tbody>
</table>
Signaling Games

- models simplest case of information flow between two agents
- sender has info, sends message, receiver reacts
- originally to account for evolution of linguistic conventions (Lewis, 1969)
- but also many others:
  - economics: Spence (1973)
  - biology: Grafen (1990)
  - pragmatics: Parikh (1991)
- overview on signaling games:
  - Sobel (2008)
  - Skyrms (2010)
Signaling Games

\[ t_s \in T \quad m \in M \quad t_r \in T \]

\[ t_s = t_r \quad \uparrow \quad \text{success} \]
Signaling Games

$t_s \in T \quad m \in M \quad t_r \in T$

$t_s \neq t_r \quad \updownarrow \quad \text{failure}$
Signaling Games — Informal Characterization

- fix set of states $T$
- each $t \in T$ occurs with some prior probability $\Pr(t)$
- sender $S$ knows actual state $t \in T$
- receiver $R$ doesn’t but knows prior distribution $\Pr \in \Delta(T)$
- $S$ chooses a message $m \in M$
- $R$ observes $m$ and chooses an action $a \in A$
- both $S$ and $R$ receive payoffs depending on $t$, $m$ and $a$
Interpretation Game for DoPL
Strategy Profiles for DoPL-Game

Horn Convention

Unstable Pattern

Anti-Horn Convention

Smolensky Convention
Strategy Profiles for DoPL-Game

Horn Convention

Unstable Pattern

Anti-Horn Convention

Smolensky Convention
Strategy Profiles for DoPL-Game

Horn Convention

Anti-Horn Convention

Unstable Pattern

Smolensky Convention
Strategy Profiles for DoPL-Game

Horn Convention

Unstable Pattern

Anti-Horn Convention

Smolensky Convention
Pure Strategy Profiles of the DoPL-Game
Nash Equilibria of the DoPL-Game

1

2

3

4

5

6

7

8

9

10

11

12

13

14

15

16
Challenge for Game Theoretic Pragmatics

1. construct/motivate interpretation games
2. single out desired solution with adequate solution concept
IBR Model

1. Game Model
   - represents context of utterance
   - uniquely determines IBR Reasoning

2. IBR Reasoning
   - captures reasoning about language use
   - uniquely determines Output / Prediction

Output / Prediction:
- optimal speaker-hearer behavior

Input:
- (i) to-be-interpreted utterance & (ii) its alternative expressions

Empirical Data:
- (i) trained intuitions & (ii) psycholinguistic data
Iterated Best Response Reasoning

a model of stepwise, hypothetical reasoning:

(i) player $i$ assumes that player $j$ does $X$ \textit{(focal starting point)}
(ii) then player $i$ considers best response $Y$ to $X$
(iii) then player $i$ considers player $j$’s best response $X'$ to $Y$
(iv) …
(v) terminate when looping

Idea: Focality of Semantic Meaning

pragmatic reasoning starts from the semantics
IBR Reasoning for Signaling Games

- Sends any true message
- Best response to $S_0$
- Best response to $R_0$
- Best response to $S_1$
- Best response to $R_1$
- Best response to $S_2$
- Best response to $R_2$
- Best response to $S_3$
- Best response to $R_3$
- Interprets messages literally
Naïve Receiver

\[ m \rightarrow t \]

\[ m^* \rightarrow t^* \]

Level-1 Sender

\[ t \rightarrow m \]

\[ t^* \rightarrow m^* \]

Level-2 Receiver

\[ m \rightarrow t \]

(forward induction)

\[ m^* \rightarrow t^* \]

Level-3 Sender

\[ t \rightarrow m \]

\[ \leadsto \text{fixed point} \]

\[ t^* \rightarrow m^* \]
Naïve Receiver

\[ m \rightarrow t \]
\[ m^* \rightarrow t^* \]

Level-1 Sender

\[ t \rightarrow m \]
\[ t^* \rightarrow m^* \]

Level-2 Receiver

\[ m \rightarrow t \]
\[ m^* \rightarrow t^* \]

(forward induction)

Level-3 Sender

\[ t \rightarrow m \]
\[ t^* \rightarrow m^* \]

\[ \rightsquigarrow \text{fixed point} \]
Naïve Receiver

Level-1 Sender

Level-2 Receiver

(forward induction)

Level-3 Sender

\[ m \rightarrow t \]

\[ m^* \rightarrow t^* \]

\[ t \rightarrow m \]

\[ t^* \rightarrow m^* \]

\[ m \rightarrow t \]

\[ m^* \rightarrow t^* \]

\[ t \rightarrow m \]

\[ t^* \rightarrow m^* \]

\[ \rightsquigarrow \text{fixed point} \]
**Naïve Receiver**

\[ m \rightarrow t \]

\[ m^* \rightarrow t^* \]

**Level-1 Sender**

\[ t \rightarrow m \]

\[ t^* \rightarrow m^* \]

**Level-2 Receiver**

\[ m \rightarrow t \]

(forward induction)

\[ m^* \rightarrow t^* \]

**Level-3 Sender**

\[ t \rightarrow m \]

\[ t^* \rightarrow m^* \]

\[ \rightsquigarrow \text{fixed point} \]
Naïve Receiver

\[ m \rightarrow t \]

\[ m^* \rightarrow t^* \]

Level-1 Sender

\[ t \rightarrow m \]

\[ t^* \rightarrow m^* \]

Level-2 Receiver

\[ m \rightarrow t \]

(forward induction)

\[ m^* \rightarrow t^* \]

Level-3 Sender

\[ t \rightarrow m \]

\[ t^* \rightarrow m^* \]

\[ \rightarrow \text{ fixed point} \]
Naïve Receiver

\[ m \rightarrow t \]

\[ m^* \rightarrow t^* \]

Level-1 Sender

\[ t \rightarrow m \]

\[ t^* \rightarrow m^* \]

Level-2 Receiver

\[ m \rightarrow t \]

(forward induction)

\[ m^* \rightarrow t^* \]

Level-3 Sender

\[ t \rightarrow m \]

\[ t^* \rightarrow m^* \]

\[ \rightarrow \text{fixed point} \]
Relation BiOT & IBR

(Franke and Jäger, 2011)

- not equivalent, but almost
- main difference: IBR captures quantity reasoning (scalar implicatures), BiOT only does when proper constraints are given
Relation BiOT & Reinforcement Learning  (Franke and Jäger, 2011)

<table>
<thead>
<tr>
<th>$S$’s choice</th>
<th>$m$</th>
<th>$m^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>.7</td>
<td>.3</td>
</tr>
<tr>
<td>$t^*$</td>
<td>.7</td>
<td>.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$R$’s choice</th>
<th>$t$</th>
<th>$t^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>.7</td>
<td>.3</td>
</tr>
<tr>
<td>$m^*$</td>
<td>.7</td>
<td>.3</td>
</tr>
</tbody>
</table>
Relation BiOT & Reinforcement Learning  
(Franke and Jäger, 2011)

\[
\begin{array}{ccc}
\text{BiOT-System} \\
\hline
S'\text{'}s \text{ choice} & m & m^* \\
\hline
\text{} & \text{.8} & \text{.2} \\
\text{} & \text{.6} & \text{.4} \\
\hline
\end{array}
\]

\[
\begin{array}{ccc}
R'\text{'}s \text{ choice} & t & t^* \\
\hline
\text{m} & \text{.8} & \text{.2} \\
\text{m}^* & \text{.6} & \text{.4} \\
\hline
\end{array}
\]
Relation BiOT & Reinforcement Learning  (Franke and Jäger, 2011)

\[
\begin{array}{ccc}
\text{S’s choice} & m & m^* \\
\hline
 t & .9 & .1 \\
 t^* & .5 & .5 \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{R’s choice} & t & t^* \\
\hline
 m & .9 & .1 \\
 m^* & .5 & .5 \\
\end{array}
\]
Relation BiOT & Reinforcement Learning  
(Franke and Jäger, 2011)

BiOT-System

<table>
<thead>
<tr>
<th>S’s choice</th>
<th>$m$</th>
<th>$m^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$t^*$</td>
<td>.4</td>
<td>.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>R’s choice</th>
<th>$t$</th>
<th>$t^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$m^*$</td>
<td>.4</td>
<td>.6</td>
</tr>
</tbody>
</table>
Relation BiOT & Reinforcement Learning  (Franke and Jäger, 2011)

- weak optimality $\approx$ most likely path of reinforcement learning
  
  \textbf{NB:} tight connection RL-learning & replicator dynamics
  
  (Börgers and Sarin, 1997)

- parallel is close but there are divergences:
  - quantity reasoning (as before)
  - BiOT makes no arbitrary meaning enrichment, RL might
Conclusions

- exact interpretation of \textsc{biot} still open
- \textsc{ibr} and \textsc{rl} come close
- upshot of comparison
  - \textsc{biot} is very top-level
  - more concrete reasoning/evolution schemes show limitations
References


