Game Theoretic Pragmatics

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Game Theoretic Pragmatics ...

- aims for mathematically precise models of language use and interpretation ...
- by formally representing interlocutors':
  (i) action alternatives
  (ii) preferences
  (iii) beliefs about (i)–(iii).

Today's Agenda

- introduce game theory
- charter how to apply GT to linguistic pragmatics
  - $\rightsquigarrow$ IBR-model
- compare BIOTwith IBR-model and reinforcement learning
  - $\rightsquigarrow$ find an interpretation of strong/weak optimality

Running Example: Division of Pragmatic Labor

- Horn (1984)
- a.k.a. M-Implicature (Levinson, 2000)
- unmarked form pairs with unmarked meaning $m \leftrightarrow t$
- marked form pairs with marked meaning $m^* \leftrightarrow t^*$

Example 1 (Black Bart)

(1) a. Black Bart killed the sheriff. m

b. $\rightsquigarrow$ Black Bart killed the sheriff in a stereotypical way. t

(2) a. Black Bart caused the sheriff to die. m^*

b. $\rightsquigarrow$ BB killed the sheriff in a non-stereotypical way. t^*

Running Example: Division of Pragmatic Labor

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Example 1 (Black Bart)

(3) a. Black Bart killed the sheriff. m

b. $\rightsquigarrow$ Black Bart killed the sheriff in a stereotypical way. t

(4) a. Black Bart caused the sheriff to die. m^*

b. $\rightsquigarrow$ BB killed the sheriff in a non-stereotypical way. t^*

Example 2 (Sue's Smile)

(5) a. Sue smiled. m

b. $\rightsquigarrow$ Sue smiled genuinely. t

(6) a. The corners of Sue's lips turned slightly upwards. m^*

b. $\rightsquigarrow$ Sue faked a smile. t^*

Example 3 (Mrs T's Song)

(7) a. Mrs T sang 'Home Sweet Home.' m

b. $\rightsquigarrow$ Mrs T sang a lovely song. t

(8) a. Mrs T produced a series of sounds roughly corresponding to the score of 'Home Sweet Home.' m^*

b. $\rightsquigarrow$ Mrs T sang very badly. t^*

Example 2 (Sue's Smile)

(9) a. Sue smiled. m

b. $\rightsquigarrow$ Sue smiled genuinely. t

(10) a. The corners of Sue's lips turned slightly upwards. m^*

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Example 3 (Mrs T's Song)

(11) a. Mrs T sang 'Home Sweet Home.' m

b. $\rightsquigarrow$ Mrs T sang a lovely song. t

(12) a. Mrs T produced a series of sounds roughly corresponding to the score of 'Home Sweet Home.' m^*

b. $\rightsquigarrow$ Mrs T sang very badly. t^*

BIOT's Explanation

$$\begin{array}{ccc} & t & t^* \\ m & \bullet & \longleftarrow \bullet \\ & \uparrow & \uparrow \\ m^* & \bullet & \longleftarrow \bullet \end{array}$$

generator
& preferences

$$\begin{array}{ccc} & t & t^* \\ m & \odot & \times \\ m^* & \times & \bullet \end{array}$$

strong optimality
& blocking

$$\begin{array}{ccc} & t & t^* \\ m & \odot & \times \\ m^* & \times & \odot \end{array}$$

strong optimality
& weak optimality

explanation:

$\bullet$	:	possible
$\odot$	:	optimal
$\times$	:	blocked

BiOT's Explanation

$$\begin{tikzcd} & t & t^* \\ m & \bullet & \bullet \arrow[l] \\ m^* & \bullet \arrow[u] & \bullet \arrow[l] \arrow[u] \end{tikzcd}$$

generator
& preferences

	t	t^*
m	$\odot$	$\times$
m^*	$\times$	$\bullet$

strong optimality
& blocking

	t	t^*
m	$\odot$	$\times$
m^*	$\times$	$\odot$

strong optimality
& weak optimality

explanation:

- $\bullet$: possible
- $\odot$: optimal
- $\times$: blocked

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generator
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	t	t^*
m	$\odot$	$\times$
m^*	$\times$	$\bullet$

strong optimality
& blocking

	t	t^*
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strong optimality
& weak optimality

explanation:

$\bullet$	:	possible
$\odot$	:	optimal
$\times$	:	blocked

Controversial Issue

how to interpret strong/weak optimality?

1. online reasoning? (e.g. Hendriks et al., 2010)
2. diachronic optimization? (e.g. Blutner and Zeevat, 2008)


Dungeons & Dragons
Rules Cyclopedia
GET READY!
Sagaland
Clue
MASTER MIND
CRANIUM
MONOPOLY
GO
ROBO RALLY
RISK
The Game of Global Domination
BATTLETECH
A GAME OF ARMORED COMBAT
DOUGLAS AND GRAHAM WALKER
SETTLERS
CATAN
ROCK PAPER SCISSORS
WARHAMMER
BATTLE FOR SKULL PASS

Branches of Game Theory

- classical game theory (since 1940)
  - (mostly) assumes perfectly rational agents
  - central notion: Nash equilibrium
- evolutionary game theory (since 1970)
  - long-term optimization of boundedly-rational agents
  - central notions: evolutionary stability & replicator dynamics
- behavioral game theory (since 1990)
  - studies interactive decision making in the lab
  - seeks regularities in choices beyond perfect rationality

Let's Play: p-Beauty Contest (with $p = {}^2/_3$)

Everybody choose and write down a number from 0 to 100 (including each). We will sum and average all choices. The person(s) closest to ${}^2/_3$ of the average will win.

Kinds of Games

uncertainty	choice points	
	simultaneous	in sequence
no	strategic/static	dynamic/sequential with complete info
yes	Bayesian	dynamic/sequential with incomplete info

Games vs. Solutions

- Game Models:
  representations of a choice situation
- Solutions Concepts:
  capture *particular* behavior:
  good, optimal, rational, stable (...)

Static Games

- players choose simultaneously
- players have complete information

Examples

	a_c	a_d
a_c	2,2	0,3
a_d	3,0	1,1

Prisoner's Dilemma

	a_{stay}	a_{go}
a_{stay}	2,2	0,0
a_{go}	0,0	1,1

Coordination Problem

Static Games

- players choose simultaneously
- players have complete information

Examples

	a_c	a_d
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Prisoner's Dilemma

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a_{stay}	2,2	0,0
a_{go}	0,0	1,1

Coordination Problem

Nash Equilibrium (Intuition)

Arrangement of strategies, one for each player, such that no player would benefit from unilateral deviation (i.e., no player would be better off doing something else if everybody else keeps doing the same thing).

Example (Prisoner's Dilemma)

	a_c	a_d
a_c	2,2	0,3
a_d	3,0	1,1

- single pure NE: $\langle a_d, a_d \rangle$

Example (Coordination)

	a_{stay}	a_{go}
a_{stay}	2,2	0,0
a_{go}	0,0	1,1

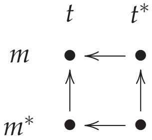
- two pure NEs: $\langle a_{\text{stay}}, a_{\text{stay}} \rangle$, and $\langle a_{\text{go}}, a_{\text{go}} \rangle$

Interpretation Game for DoPL (Dekker and van Rooij, 2000)

$$\begin{tikzcd}
 & t & t^* \\
m & \bullet & \bullet \arrow[l] \\
m^* & \bullet \arrow[u] & \bullet \arrow[l] \arrow[u]
\end{tikzcd}$$

BiOT-System

Interpretation Game for DoPL (Dekker and van Rooij, 2000)



BiOT-System

	t	t^*
m	2,2	1,1
m^*	1,1	0,0

Static Game

Interpretation Game for DoPL (Dekker and van Rooij, 2000)

$$\begin{array}{ccc} & t & t' \\ m & \odot & \times \\ m' & \times & \bullet \end{array}$$

Strong Optimality

	t	t^*
m	2,2	1,1
m^*	1,1	0,0

Static Game

Interpretation Game for DoPL (Dekker and van Rooij, 2000)

	t	t'
m	$\odot$	$\times$
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Strong Optimality

	t	t^*
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Nash Equilibrium

Interpretation Game for DoPL (Dekker and van Rooij, 2000)

	t	t'
m	$\odot$	$\times$
m'	$\times$	$\odot$

Weak Optimality

	t	t^*
m	2,2	1,1
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Nash Equilibrium

Interpretation Game for DoPL (Dekker and van Rooij, 2000)

$$\begin{array}{ccc} & t & t' \\ m & \odot & \times \\ m' & \times & \odot \end{array}$$

Weak Optimality

	t	t^*
m	2,2	1,1
m^*	1,1	0,0

???

Static Games & BiOT (Dekker and van Rooij, 2000)

- BiOT-Systems $\leftrightarrow$ Static Games
- strong optimality $\leftrightarrow$ Nash equilibrium
- weak optimality $\leftrightarrow$ iterated Nash equilibrium (???)

Criticism

- static interpretation game means that:
  1. speaker and hearer choose simultaneously
  2. speaker chooses utterance independently of meaning that she wants to express
  3. hearer chooses interpretation independently of any utterance that needs to be interpreted

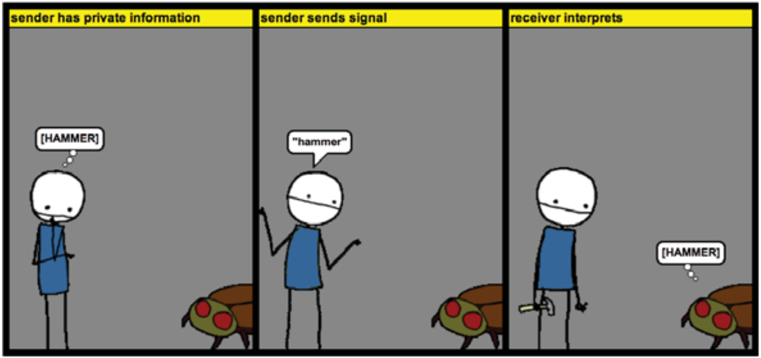
Kinds of Games

uncertainty	choice points	
	simultaneous	in sequence
no	*strategic / static*	*dynamic / sequential* with *complete* info
yes	*Bayesian*	*dynamic / sequential* with *incomplete* info $\bigcup$ signaling games $\bigcup$ interpretation games

Signaling Games

- models simplest case of information flow between two agents
- sender has info, sends message, receiver reacts
- originally to account for evolution of linguistic conventions (Lewis, 1969)
- but also many others:
  - economics: Spence (1973)
  - biology: Grafen (1990)
  - pragmatics: Parikh (1991)
- overview on signaling games:
  - Sobel (2008)
  - Skyrms (2010)

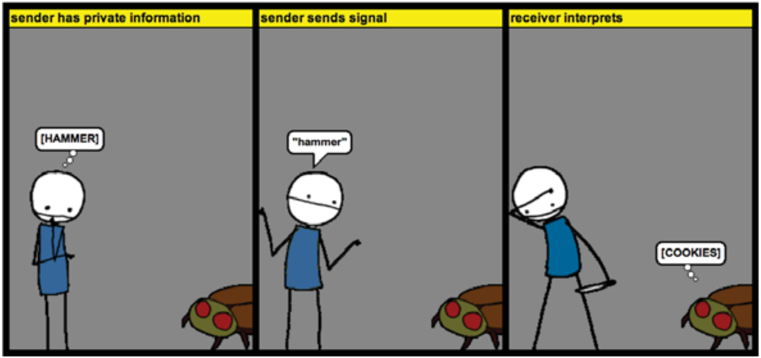
Signaling Games



$t_s \in T$ $\quad m \in M$ $\quad t_r \in T$

$$t_s = t_r$$
$$\Updownarrow$$
success

Signaling Games

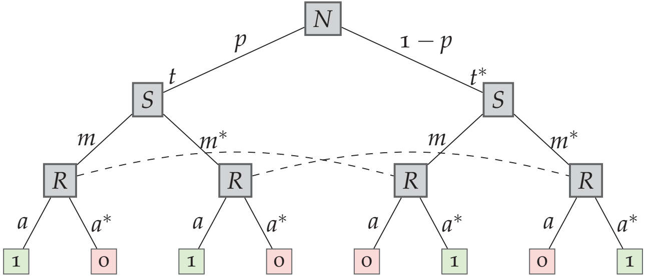


$t_s \in T$ $m \in M$ $t_r \in T$

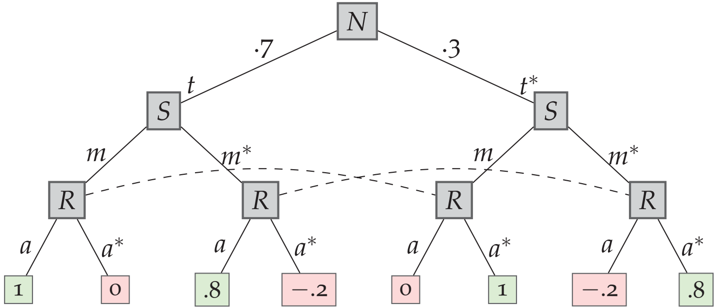
$t_s \neq t_r$
$\Updownarrow$
failure

Signaling Games — Informal Characterization

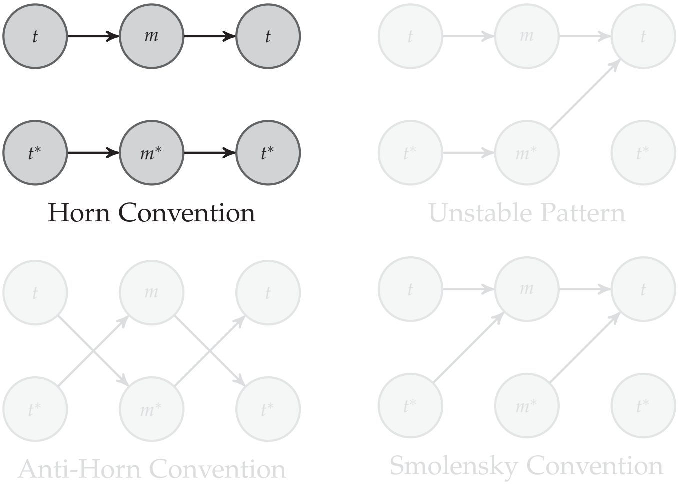
- fix set of states T
- each $t \in T$ occurs with some *prior probability* $\Pr(t)$
- sender S knows *actual state* $t \in T$
- receiver R doesn't but knows *prior distribution* $\Pr \in \Delta(T)$
- S chooses a message $m \in M$
- R observes m and chooses an action $a \in A$
- both S and R receive payoffs depending on t, m and a



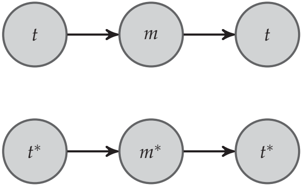
Interpretation Game for DoPL



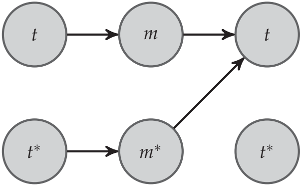
Strategy Profiles for DoPL-Game



Strategy Profiles for DoPL-Game



Horn Convention

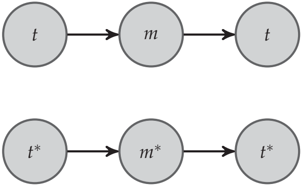


Unstable Pattern

Anti-Horn Convention

Smolensky Convention

Strategy Profiles for DoPL-Game



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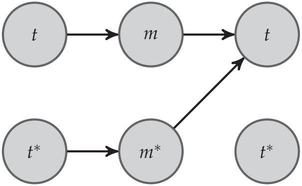

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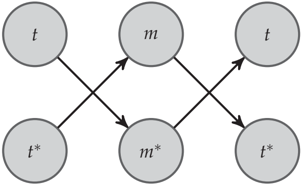
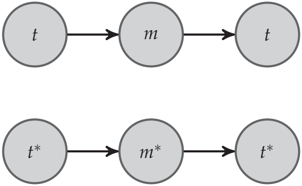


Anti-Horn Convention

Smolensky Convention

Strategy Profiles for DoPL-Game



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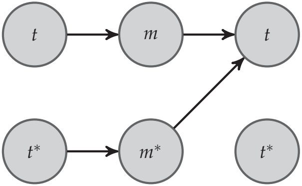

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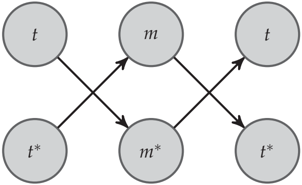
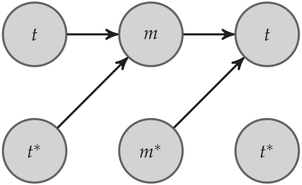
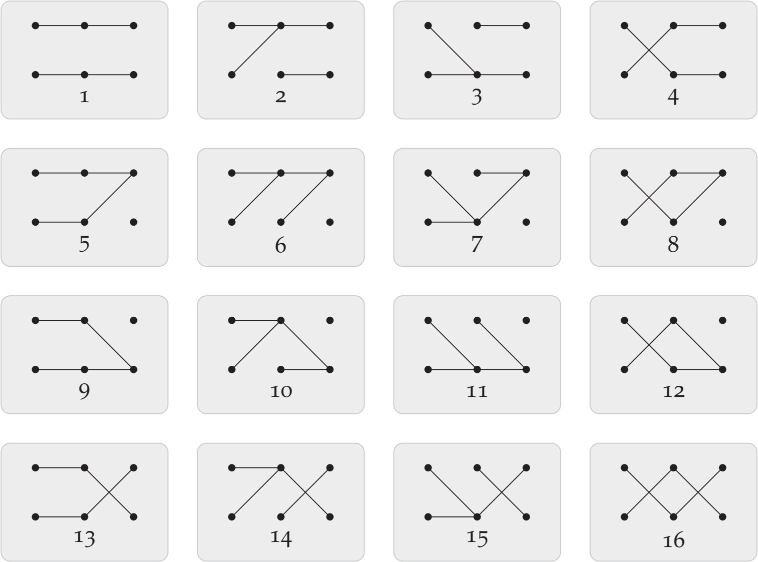


Anti-Horn Convention

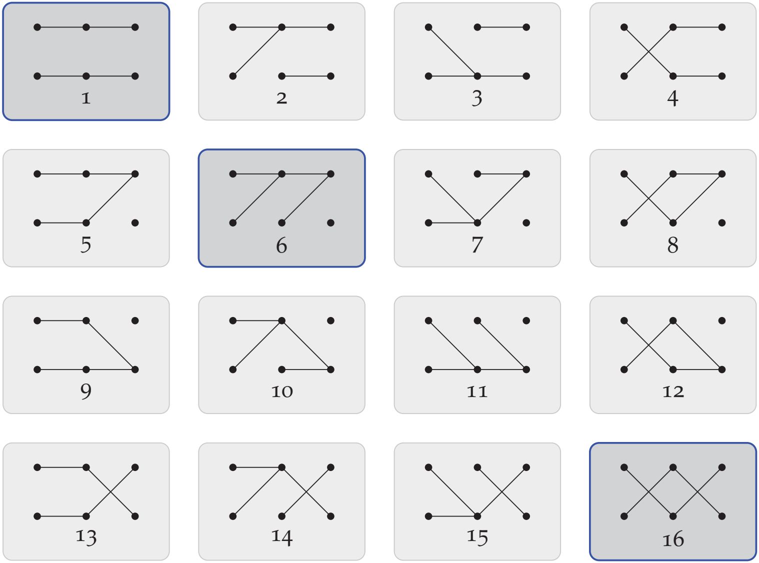


Smolensky Convention

Pure Strategy Profiles of the DoPL-Game

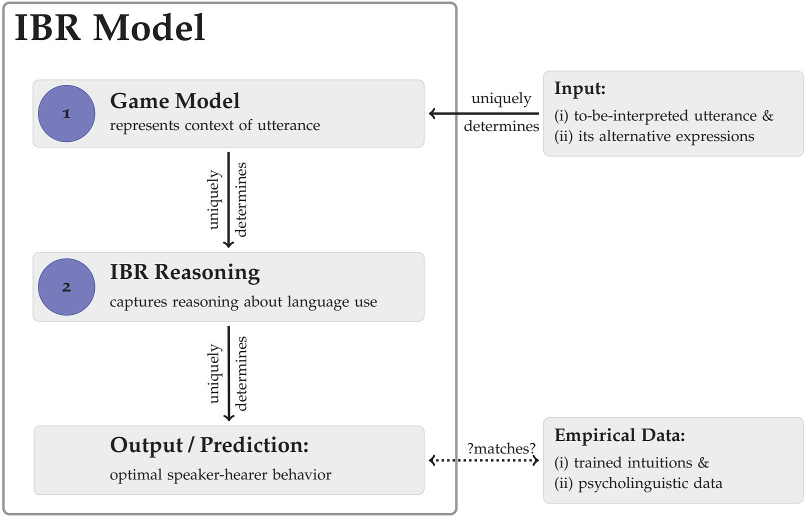


Nash Equilibria of the DoPL-Game



Challenge for Game Theoretic Pragmatics

1. construct/motivate interpretation games
2. single out desired solution with adequate solution concept


IBR Model
1
Game Model
represents context of utterance
uniquely determines
2
IBR Reasoning
captures reasoning about language use
uniquely determines
Output / Prediction:
optimal speaker-hearer behavior
Input:
(i) to-be-interpreted utterance &
(ii) its alternative expressions
uniquely determines
Empirical Data:
(i) trained intuitions &
(ii) psycholinguistic data
?matches?

Iterated Best Response Reasoning

a model of stepwise, hypothetical reasoning:

(i) player i assumes that player j does X (focal starting point)

(ii) then player i considers *best response* Y to X

(iii) then player i considers player j's best response X' to Y

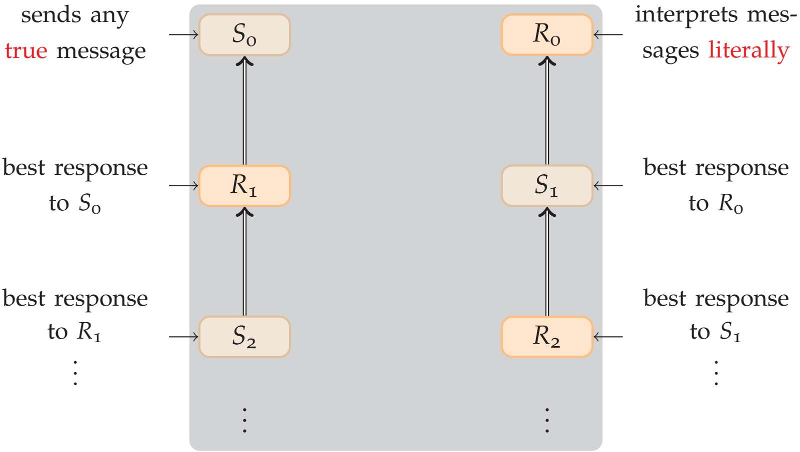
(iv) ...

(v) terminate when looping

Idea: Focality of Semantic Meaning

pragmatic reasoning starts from the semantics

IBR Reasoning for Signaling Games



Naïve Receiver

$$
\begin{tikzcd}
m \arrow[r] & t \\
m^* \arrow[ur] & t^*
\end{tikzcd}
$$

Level-2 Receiver

$$
\begin{tikzcd}
m \arrow[r] & t \\
m^* \arrow[r] & t^*
\end{tikzcd}
$$

(forward induction)

Level-1 Sender

$$
\begin{tikzcd}
t \arrow[r] & m \\
t^* \arrow[ur] & m^*
\end{tikzcd}
$$

Level-3 Sender

$$
\begin{tikzcd}
t \arrow[r] & m \\
t^* \arrow[r] & m^*
\end{tikzcd}
$$

$\leadsto$ fixed point

Naïve Receiver

$$\begin{tikzcd} m \arrow[r] & t \\ m^* \arrow[ur] & t^* \end{tikzcd}$$

Level-1 Sender

$$\begin{tikzcd} t \arrow[r] & m \\ t^* \arrow[ur] & m^* \end{tikzcd}$$

Level-2 Receiver

$$\begin{tikzcd} m \arrow[r] & t \\ m^* \arrow[r] & t^* \end{tikzcd}$$

(forward induction)

Level-3 Sender

$$\begin{tikzcd} t \arrow[r] & m \\ t^* \arrow[r] & m^* \end{tikzcd}$$

$\rightsquigarrow$ fixed point

Naïve Receiver

$$\begin{tikzcd} m \arrow[r] & t \\ m^* \arrow[ur] & t^* \end{tikzcd}$$

Level-1 Sender

$$\begin{tikzcd} t \arrow[r] & m \\ t^* \arrow[ur] & m^* \end{tikzcd}$$

Level-2 Receiver

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(forward induction)

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(forward induction)

Level-3 Sender

$$\begin{tikzcd} t \arrow[r] & m \\ t^* \arrow[r] & m^* \end{tikzcd}$$

$\rightsquigarrow$ fixed point

Relation BiOT & IBR (Franke and Jäger, 2011)

- not equivalent, but almost
- main difference: IBR captures quantity reasoning (scalar implicatures), BiOT only does when proper constraints are given

Relation BiOT & Reinforcement Learning (Franke and Jäger, 2011)

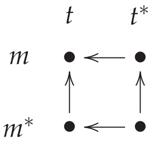
$$\begin{tikzcd}[row sep=large, column sep=large]
 & t & t^* \\
m & \bullet & \bullet \arrow[l] \\
m^* & \bullet \arrow[u] & \bullet \arrow[l] \arrow[u]
\end{tikzcd}$$

BiOT-System

S's choice	m	m^*
t	.7	.3
t^*	.7	.3

R's choice	t	t^*
m	.7	.3
m^*	.7	.3

Relation BiOT & Reinforcement Learning (Franke and Jäger, 2011)



BiOT-System

S's choice	m	m^*
t	.8	.2
t^*	.6	.4

R's choice	t	t^*
m	.8	.2
m^*	.6	.4

Relation BiOT & Reinforcement Learning (Franke and Jäger, 2011)

$$\begin{tikzcd}
 & t & t^* \\
m & \bullet & \bullet \arrow[l] \\
m^* & \bullet \arrow[u] & \bullet \arrow[l] \arrow[u]
\end{tikzcd}$$

BiOT-System

S's choice	m	m^*
t	.9	.1
t^*	.5	.5

R's choice	t	t^*
m	.9	.1
m^*	.5	.5

Relation BiOT & Reinforcement Learning (Franke and Jäger, 2011)

$$\begin{tikzcd} & t & t^* \\ m & \bullet & \bullet \arrow[l] \\ m^* & \bullet \arrow[u] & \bullet \arrow[l] \arrow[u] \end{tikzcd}$$

BiOT-System

S's choice	m	m^*
t	1	0
t^*	.4	.6

R's choice	t	t^*
m	1	0
m^*	.4	.6

Relation BiOT & Reinforcement Learning (Franke and Jäger, 2011)

- weak optimality ≈ most likely path of reinforcement learning

  NB: tight connection RL-learning & replicator dynamics (Börgers and Sarin, 1997)
- parallel is close but there are divergences:
  - quantity reasoning (as before)
  - BIOT makes no arbitrary meaning enrichment, RL might

Conclusions

- exact interpretation of BiOT still open
- IBR and RL come close
- upshot of comparison
  - BiOT is very top-level
  - more concrete reasoning/evolution schemes show limitations

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