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# The Potential of Quantum Probability for Modeling Cognitive Processes

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## General Background and Speakers

Quantum probability (QP) theory is a theory for how to assign probabilities to observables. In the context of physics, it has been successfully employed by researchers for over 100 years and has been the basis for some of the most impressive discoveries of the human mind (e.g., the transistor, and so the microchip, and the laser). But the applicability of QP theory is not limited to physical phenomena and, indeed, there has been growing interest in exploring the potential of QP theory in areas as diverse as economics (Baaquie, 2004), information theory (e.g., Grover, 1997), and psychology.

We are interested in the latter objective. Probabilistic approaches to cognition have been recently enjoying considerable success (e.g., Griffiths et al., 2010). This is perhaps unsurprising. Several (if not most) aspects of cognitive processing involve inference about quantities which cannot be determined with certainty. Thus, probability theory seems an ideal tool for formalizing uncertainty in cognitive processes and describing the mechanisms which allow humans to be successful in an uncertain world. The question then becomes to identify the probabilistic framework which is most suitable for modeling particular aspects of cognition.

Cognitive scientists have widely employed classical probability theory and there is no doubt that such approaches can be incredibly successful for particular problems (e.g., Oaksford & Chater, 2007). But is Bayesian probability *always* the most suitable probabilistic framework for describing cognitive processes?

Human inference often appears to violate fundamental laws of classical probability. For example, in the famous Linda problem of Tversky and Kahneman (1983) participants consider as more likely a conjunctive statement than a corresponding single premise one (i.e., *Probability* (*A*&*B*) > *Probability* (*A*); this is the conjunction fallacy). In one-shot prisoner's dilemma games, naïve participants choose to defect knowing their opponent defects, they choose to also defect knowing their opponent cooperates, but they reverse their judgment and choose to cooperate when their opponent's action is unknown (Shafir & Tversky, 1992). This result is also hard to reconcile with classical probability theory, it violates the sure thing principle and the law of total probability. Findings such as the conjunction fallacy and the violation of the sure thing principle provide the motivation for exploring probabilistic frameworks alternative to classic probability theory.

QP theory is mostly consistent with classical probability theory, but for a crucial difference. In classical probability theory *Probability* (*A*&*B*) = *Probability* (*B*&*A*), but QP theory does not always obey commutativity in conjunction. In QP theory probability assessment can be order (or context) dependent. Relatedly, QP theory is not constrained by the law of total probability (though it is subject to other constraints). The law of total probability in classical theory can be expressed as *Probability* (*A*) = *Probability* (*A*&*X*) + *Probability* (*A* & not *X*), but this decomposition in QP theory may involve interference terms. As we have shown (e.g., Busemeyer et al., in press; Busemeyer, Wang, & Townsend, 2006; Pothos & Busemeyer, 2009), these properties of QP theory can lead to successful and simple QP models for empirical findings, such as the conjunction fallacy and the violation of the sure thing principle.

Finally, QP theory remains suitable for describing cognition without involving any claim that the brain is a quantum computer. Rather, we adopt what is a standard approach in cognitive science and ask: 'does quantum probability theory produce results which predict human behavior well?'

The application of QP theory in cognition holds a lot of promise. The main objective of this symposium is to present the key ideas to the cognitive science community and summarize progress with QP theory cognitive models. What is the potential of such models? How well do they cover the relevant empirical findings? What are the key differences between QP theory models and matched classical probability ones? Three talks will address these issues (Pothos et al., Blutner, and Atmanspacher). Equally, the symposium will be an opportunity to consider concerns with the applicability of QP theory. The fourth talk will cast a critical eye and address the issue of the complexity of QP models and their comparability with standard approaches.

## Overview of Talks

### QP theory in decision making

**Pothos, Busemeyer, Trueblood, & Wang.** Pothos is a cognitive scientist with a background in physics. Trueblood, Wang, and Pothos have all been working with Busemeyer on QP models. Busemeyer is the mathematical psychologist who first presented to the cognitive science community detailed, formal models for psychological processes based on QP theory. The talk will review the successful application of QP theory in three important empirical situations: the conjunction fallacy in Linda-type problems, order effects in human inference, and violations of the sure thing principle in Prisoner's Dilemma.

### QP theory models of semantic composition

**Blutner.** This speaker is a theoretical physicist by training, interested in not only cognitive modeling, but also artificial intelligence and philosophy. He will consider a sophisticated geometrical approach for semantic composition. For example, what determines the meaning of "red nose" or "red flag"? His model will provide a solution for Quine's problem of determining which part of an apple or grapefruit has to be red in order to call it a "red apple" or "reddish grapefruit". The proposed quantum model is based on tensor products and entangled states expressing the results of conceptual combinations. The proposal conforms to Millikan's and Recanati's view that comprehending language is normally as direct as the perception of visual scenery. In both cases, beliefs are formed in a direct way, apparently without reference to serial processes of inferential interpretation.

### A generalized quantum framework for bistable perception

**Atmanspacher.** This speaker, trained in theoretical physics, is the Head of the Theory and Data Analysis department at the Institute for Frontier Areas of Psychology at Freiburg. He has pioneered the development of a formal framework, based on those elements of quantum theory which are considered pertinent for cognitive modeling, such as complementarity, whose historical origin can, in fact, be traced back to psychology. He will present an application for the perception of ambiguous stimuli, such as the Necker cube. The perception process is modeled as the evolution of an unstable two-state system, giving rise to a "Necker-Zeno" effect, generalizing the quantum Zeno effect to mental systems. Quantitative relations between the involved time scales are theoretically derived and found to be consistent with empirical observation. Moreover, the Necker-Zeno model predicts phenomena of temporal nonlocality, referring to mental states that are not sharply localized in time but extend over a time interval of non-zero duration.

### Model Selection applied to QP models

**Shiffrin & Busemeyer.** Shiffrin is a cognitive psychologist with many seminal contributions to cognitive theory. With Busemeyer, he will discuss the suitability of QP theory for cognitive modeling and consider some practical issues regarding the way QP models can be compared to traditional probability models. Do QP models fit better simply because they are more complex? Do traditional methods for model comparison such as Bayesian Model Selection, Minimum Description Length, and Cross Validation pose special problems when employed for quantum probability models? These and other model selection issues will be addressed. Simulation results will be presented for some simple cases to illustrate the situation and point to potential solutions.

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