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KINEMATICAL EFFECTS IN THE MULTI-REGGE MODEL AND APPROACH TO SCALING IN THE CENTRAL REGION

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Abstract: The careful treatment of the kinematics of the MRM in the central region leads to an increase of the structure function towards its asymptotic value like $a + b\mu/\sqrt{s}$ where μ is the transverse mass of the produced particle. This is in good agreement with experimental data on $\pi^+p \to \pi^- + ..$ from 3.7 to 18.5 GeV/c and $pp \to \pi^\pm +$. at primary momenta from 22 to 1500 GeV/c.

1. Introduction

Recently a number of experimental and theoretical investigations on inclusive single particle spectra [1–5] concentrate upon the study of the approach to the scaling limit in the pionization region ($x \approx 0$). On this question there is some theoretical controversy.

The experimental data show the following main features of the structure function $F(q, s) = E\, \mathrm{d}\sigma/\mathrm{d}q$ (here as usual s is the square of the c.m. total energy and E, q the are energy and momentum of the observed particle in the c.m.s.)·

(i) As a function of the rapidity $y = \tanh^{-1}(q_{\parallel}/E)$ the distribution develops a plateau around $y = 0$. This central plateau is approached from below.

(ii) The s-dependence of the deviation from the scaling limit

$$\frac{\Delta F(q, s)}{F_{\mathrm{sc}}(q, s)} = \frac{F(q,s) - F_{\mathrm{sc}}}{F_{\mathrm{sc}}}$$

is still rather uncertain. According to Ferbel [5] the function $-\mathrm{const.}\, s^{-\frac{1}{4}}$ gives a good fit for $\Delta F(x,s)/F_{\mathrm{sc}}(x)$ at $x = 0$ where $x = 2q_{\parallel}/\sqrt{s}$ is the Feynman scaling variable [6]. Meyer and Struczinski [7] have concluded that const. $s^{-\frac{1}{2}}$ gives a more suitable description for the approach to scaling. We have found that the data reviewed by Lillethun [8] do not allow one to decide between these possibilities.

(iii) At fixed energies the magnitude of the deviation from the scaling limit increases with growing mass of the observed particle. This is observed especially com paring the reactions pp → $\bar{p}$ + anything, pp → K^- + anything, pp → π^- + anything [9]. For a particular observed particle the deviations from the scaling limit increase with transverse momentum $q_\perp$ as the data from the reaction pp → $\pi^\pm$ + anything [8] show. These two points suggested that the function $\Delta F/F_{sc}$ at fixed s increases with the transverse mass $\mu = \sqrt{m^2 + q_\perp^2}$ of the observed particle.

Concerning these three features of the approach to scaling in the central region various models for inclusive reactions make different predictions. The thermodynamic model [10] with the strong bootstrap solution [11] and a slight energy dependent fireball velocity weight function gives scaling behaviour and a central plateau in the rapidity [1, 11]. In the central region the scaling limit is approached from below due to the increasing mass of the fireballs with the primary energy. The predicted approach is in quantitative agreement with experimental data [1, 2]. The deviation from the scaling limit $|\Delta F/F_{sc}|$ at fixed s increases with the transverse mass μ [1].

In the work by Amati, Caneschi and Testa [12] the approach to scaling in the pionization region has been considered explicitely for large $q_\perp$. Using the multiperipheral model in its original version [13] the authors find the approach to scaling from below.

The usual Mueller-Regge analysis based on the generalized optical theorem [14] gives scaling from above as $s^{-\frac{1}{4}}$ in the central region if the pomeron (P) and meson (M) trajectories are included. In the work of Chan et al. [3] it has been shown that the approach to scaling from below necessitates the introduction of a further singularity Q with negative residue.

The various inclusive multi-Regge models differ with regard (i) to the phase-space approximations and (ii) to the number of different trajectories which are included in the multi-Regge chain. In a one-channel model only one sort of trajectory is exchanged inside the Regge chain. A two-channel model contains two sorts of trajectories, e.g. P- and M-trajectories, where usually a zero PP -coupling is assumed. The simple Chew-Pignotti model [15] is a one-channel model, which uses a rather simplified phase space in the strong ordering limit [16] and ignores transverse momenta. This model gives a central plateau in y and an exact scaling behaviour already at finite energies [17].

More recently Pignotti and Ripa [18] have also considered the multi-Regge model for the central region using the same kinematic approximations, but they included P- and M-exchange. The calculated single particle distribution approaches a limiting rapidity plateau from above.

Tan [19] has discussed cluster formation in a two-channel model with the same phase-space approximation as used by Chew and Pignotti [15]. It is shown that the scaling limit is approached from below if the external particle-reggeon couplings are suitably chosen.

For one-channel Regge dynamics Chew et al. [20] have treated the phase space in an exact way. It is possible to express the structure function by the solution of the Chew-Goldberger-Low (CGL)-integral equation. Based on this concept Caneschi

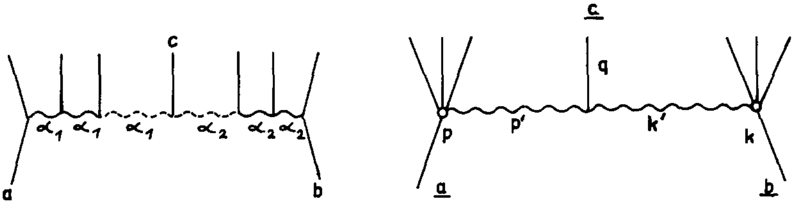

Fig. 1(a) Multi-Regge graph. (b) Notation of momenta used in the text for the double Regge graph.

and Pignotti [21] and later Silverman and Tan [22] have derived expressions for the structure function in the pionization region suitable for numerical calculations.

Caneschi [23] has argued that a model with exponential cut-off in the momentum transfers leads to an increase of the structure function from $s = 60\ \mathrm{GeV}^2$ to $s = 2000\ \mathrm{GeV}^2$. A physical understanding can be obtained by the complete harmonic analysis of the multi-peripheral inclusive distribution given by Bassetto and Toller [24]. In their expression for the inclusive distribution extra singularities occur besides the Lorentz poles that determine the absorptive part. As discussed by Caneschi [25] the negative sign of the residue of the first extra singularity arises from the exponential cut-off in the momentum transfers and leads to an approach to scaling from below with $s^{-\frac{1}{2}}$ for $x = 0$.

In this work we use the multi-Regge model by Caneschi and Pignotti [21] with exact phase space, and give a quantitative discussion of the expression for the deviation from scaling that is due to these extra singularities mentioned by Bassetto and Toller [24]. Using results of Silverman and Tan [22] we give in sect. 2 the expression for the one-particle distribution in the pionization region. In sect. 3 we derive the zeroth order term in the expansion of the structure function in powers of $s^{-\frac{1}{2}}$ and discuss the scaling limit. In sect. 4 we present our analytically calculated deviations from the scaling limit at $x = 0$ (first order term) and discuss them. In sect. 5 we compare our numerical results with data on $\pi^+ p \to \pi^\pm + \ldots$ from 22 to 1500 GeV/c [8].

2. Single particle spectrum from the multi-Regge model

The inclusive single-particle spectrum of the reaction a + b → c + anything is the sum over all exclusive multi-Regge contributions. We assume that in all multi-Regge diagrams (fig. 1a) on the left (right) of particle c, only trajectories α_1 (α_2) are exchanged. According to Silverman and Tan [22] the resulting single-particle spectrum in the pionization region is then given in terms of the solution to the CGL integral equation for the auxiliary function B:

$$F(q,s)=\frac{1}{2\lambda^{\frac{1}{2}}(s,m_{\mathrm{a}}^2,m_{\mathrm{b}}^2)}\quad \mathrm{d}^4p'\,\mathrm{d}^4k'\,\delta^4(p'+k'+q)\,B_{\mathrm{a}}(-k',p',p)$$
$$\times\,|\beta(p'^2,\omega,k'^2)|^2\,B_{\mathrm{b}}(-p',k',k), \tag{1}$$

(for the notation see fig. 1b).
Here $\lambda(x,y,z)$ is the triangle function

$$\lambda(x,y,z)=x^2+y^2+z^2-2\,(xy+xz+yz). \tag{2}$$

In the following we assume that the reggeon-reggeon coupling β does not depend on the Toller angle ω and has the simple form

$$\beta(p'^2,\omega,k'^2)=\mathrm{e}^{c_1t_1}\,\mathrm{e}^{c_2t_2}, \tag{3}$$

with

$$p'^2=t_1,\qquad k'^2=t_2.$$

Using the strong ordering approximation in the Regge amplitude (not in the phase space) one obtains the solutions B_{a} and B_{b} of the CGL equation:

$$\begin{aligned}B_{\mathrm{a}}(-k',p',p)&=\left(\frac{s_1}{s_1'}\right)^{2\alpha_1(t_1)}\mathcal{M}_{\alpha}(p',p),\\ B_{\mathrm{b}}(-p',k',k)&=\left(\frac{s_2}{s_2'}\right)^{2\alpha_2(t_2)}\mathcal{M}_{\mathrm{b}}(k',k),\end{aligned} \tag{4}$$

where

$$\begin{aligned}s_1&=(-k'+p)^2, & s_2&=(-p'+k)^2,\\ s_1'&=(p'+p)^2, & s_2'&=(k'+k)^2.\end{aligned} \tag{5}$$

$\mathcal{M}_{\mathrm{a}}$ ($\mathcal{M}_{\mathrm{b}}$) is the reggeon-particle absorptive part for the reaction

$\mathrm{a}+\alpha_1\to\mathrm{a}+\alpha_1$, $(\mathrm{b}+\alpha_2\to\mathrm{b}+\alpha_2)$.

For large s_1 (s_2) it has the form

$$\begin{aligned}\mathcal{M}_{\mathrm{a}}(p',p)&\propto s_1'^{\,\tilde{\alpha}_1(0)},\\ \mathcal{M}_{\mathrm{b}}(k',k)&\propto s_2'^{\,\tilde{\alpha}_2(0)},\end{aligned} \tag{6}$$

where $\tilde{\alpha}_1(0)$ and $\tilde{\alpha}_2(0)$ are the intercepts of the output trajectories.

Inserting eqs. (3–6) in eq. (1) we obtain:

$$F(q,s)=\frac{g}{2\lambda^{\frac{1}{2}}(s,m_a^2,m_b^2)}$$
$$\times\int d^4p'\,d^4k'\,\delta^4(p'+k'+q)\,s_1'^{\tilde{\alpha}_1(0)}\left(\frac{s_1}{s_1'}\right)^{\alpha_1(t_1)}e^{a_1t_1}$$
$$\times s_2'^{\tilde{\alpha}_2(0)}\left(\frac{s_2}{s_2'}\right)^{\alpha_2(t_2)}e^{a_2t_2}, \tag{7}$$

where g, a_1, a_2 are constants. According to Silverman and Tan [22] we introduce new integration variables (s_1, s_2, t_1, t_2). The Jacobian for this change of variables is given by

$$\int d^4p'\,d^4k'\,\delta^4(p'+k'+q)=\iint d s_1'\,d s_2'\iint\frac{dt_1\,dt_2}{16\sqrt{-\Delta_4}}\theta(-\Delta_4). \tag{8}$$

Δ_4 is the Gram determinant of the four-momenta $p+p'$, $k+k'$, p, k. This change of variables leads to

$$F(q,s)=\frac{g}{2\lambda^{\frac{1}{2}}(s,m_a^2,m_b^2)}$$
$$\times\iint d s_1'\,d s_2'\,s_1'^{\tilde{\alpha}_1(0)}\,s_2'^{\tilde{\alpha}_2(0)}\left[\iint\frac{dt_1\,dt_2}{16\sqrt{-\Delta_4}}\theta(-\Delta_4)\right.$$
$$\left.\times\left(\frac{s_1}{s_1'}\right)^{\alpha_1(t_1)}e^{a_1t_1}\left(\frac{s_2}{s_2'}\right)^{\alpha_2(t_2)}e^{a_2t_2}\right]. \tag{9}$$

The expression in the bracket is analogous to the exclusive cross section $d\sigma/ds_1\,ds_2$ for a three-body reaction (with masses $\sqrt{s_1'}$, m, $\sqrt{s_2'}$ in the final state) according to Chan et al. [26] as shown in ref. [27].

Instead of the momentum q of particle c we introduce invariant variables

$$u_1=(p-q)^2,\qquad u_2=(k-q)^2, \tag{10}$$

and use the approximation

$$s_1\approx s_1'-u_1,\qquad s_2\approx s_2'-u_2 \tag{11}$$

in the amplitude. Furthermore, we assume linear trajectories

$$\alpha_1(t_1)=\alpha_1(0)+\alpha_1'\,t_1,$$
$$\alpha_2(t_2)=\alpha_2(0)+\alpha_2'\,t_2. \tag{12}$$

Then the structure function can be expressed as the following integral

$$F(q,s) = \frac{g}{32\lambda^{\frac{1}{2}}(s, m_a^2, m_b^2)} \iint \mathrm{d}s_1' \, \mathrm{d}s_2' \, s_1'^{\tilde{\alpha}_1(0)} s_2'^{\tilde{\alpha}_2(0)}$$
$$\times \left(\frac{s_1' - u_1}{s_1'}\right)^{2\alpha_1(0)} \left(\frac{s_2' - u_2}{s_2'}\right)^{2\alpha_2(0)} I(s_1', s_2'), \tag{13}$$

with

$$I(s_1', s_2') = \iint \frac{\mathrm{d}t_1 \, \mathrm{d}t_2}{\sqrt{-\Delta_4}} \; \theta(-\Delta_4) \, \mathrm{e}^{\Omega_1 t_1} \mathrm{e}^{\Omega_2 t_2},$$
$$\Omega_1 = a_1 + 2\alpha_1' \ln \left(\frac{s_1' - u_1}{s_1'}\right), \tag{14}$$
$$\Omega_2 = a_2 + 2\,\alpha_2' \ln \left(\frac{s_2' - u_2}{s_2'}\right).$$

Chan et al. [26] have evaluated the integral (14) (eqs. (3.15) and (3.16) in ref. [26]):

$$I(s_1', s_2') = 4\pi \, \mathrm{e}^{-b} \, \mathrm{e}^{d_1 s_1'/M^2} \mathrm{e}^{d_2 s_2'/M^2}$$
$$\times \frac{1}{c} \sinh \left[c \, \lambda^{\frac{1}{2}} \left(1, \frac{s_1'}{M^2}, \frac{s_2'}{M^2}\right) \right], \tag{15}$$

with

$$M^2 = (p + k - q)^2 = s + u_1 + u_2 - m_a^2 - m_b^2 - m^2,$$
$$b = \frac{1}{2} \left\{ \Omega_1 (M^2 - u_2 - m_a^2) + \Omega_2 (M^2 - u_1 - m_b^2) \right\},$$
$$d_1 = \frac{1}{2} \left\{ \Omega_1 (M^2 + u_2 - m_a^2) + \Omega_2 (M^2 - u_1 + m_b^2) \right\},$$
$$d_2 = \frac{1}{2} \left\{ \Omega_1 (M^2 - u_2 + m_a^2) + \Omega_2 (M^2 + u_1 - m_b^2) \right\},$$
$$c = \frac{1}{2} \Big\{ \Omega_1^2 \lambda (M^2, u_2, m_a^2) + \Omega_2^2 \lambda (M^2, u_1, m_b^2)$$
$$+ 2\Omega_1 \Omega_2 [M^2 (M^2 - u_1 - u_2 - m_a^2 - m_b^2 - 2m^2$$
$$- (u_1 - m_b^2)(u_2 - m_a^2)] \Big\}^{\frac{1}{2}}. \tag{16}$$

The region of the integration in (13) is given by

$$\sqrt{s_1'} \geqslant (m_1)_{\min}, \qquad \sqrt{s_2'} \geqslant (m_3)_{\min}, \tag{17}$$

$$\lambda(1, s_1'/M^2, s_2'/M^2) \geqslant 0 \, .$$

For our analytical calculations we assume for simplitICTY $(m_1)_{\min} = (m_3)_{\min} = 0$. Integrating (13) Silverman and Tan [22] performed the change of variables

$$\frac{s_1'}{M^2} = z\,(1-x), \; \frac{s_2'}{M^2} = x(1-z), \tag{18}$$

with the Jacobian

$$\mathrm{d}\, s_1' \, \mathrm{d} s_2' = M^4 \; |1 - x - z| \, \mathrm{d}x \, \mathrm{d}z \tag{19}$$

and

$$\lambda^{\frac{1}{2}} \left(1, \frac{s_1'}{M^2}, \frac{s_2'}{M^2}\right) = |1 - x - z| . \tag{20}$$

From eqs. (12) and (15) one obtains an integral over the unit square in (x, z).

$$\begin{aligned} F(q,s) = {} & \frac{\pi\, g\, (M^2)^{2+\tilde{\alpha}_1(0)+\tilde{\alpha}_2(0)}}{16\, \lambda^{\frac{1}{2}}(s, m_a^2, m_b^2)} \int_0^1 \int_0^1 \mathrm{d}\, x \, \mathrm{d}z \\ & \times (1-x-z) \frac{\mathrm{e}^{-b}}{c} \, [z(1-x)]^{\tilde{\alpha}_1(0)} \, [x(1-z)]^{\tilde{\alpha}_2(0)} \\ & \times \left[1 + \frac{\tau_1}{z(1-x)}\right]^{2\alpha_1(0)} \left[1 + \frac{\tau_2}{x(1-z)}\right]^{2\alpha_2(0)} \\ & \times \mathrm{e}^{c(1-x-z)} \, \mathrm{e}^{d_1 z\,(1-x)} \, \mathrm{e}^{d_2 x (1-z)} , \end{aligned} \tag{21}$$

with

$$\tau_1 = -\frac{u_1}{M^2}, \qquad \tau_2 = -\frac{u_2}{M^2} \; . \tag{22}$$

Eq. (21) can be rewritten in the form

$$\begin{aligned} F(q,s) = {} & g\, \frac{\pi}{16} \frac{(M^2)^{2+\tilde{\alpha}_1(0)+\tilde{\alpha}_2(0)}}{\lambda^{\frac{1}{2}}(s, m_a^2, m_b^2)} \int_0^1 \int_0^1 \mathrm{d}x \, \mathrm{d}z \\ & \times (1 - x - z) \frac{\mathrm{e}^{c-b}}{c} \, \mathrm{e}^{-(c-d_1)z-(c-d_2)x-(d_1+d_2)xz} \\ & \times [z(1-x)]^{\tilde{\alpha}_1(0)} \, [x(1-z)]^{\tilde{\alpha}_2(0)} \end{aligned} \tag{23}$$

$$\times \left[1+\frac{\tau_1}{z(1-x)}\right]^{2\alpha_1(0)} \left[1+\frac{\tau_2}{x(1-z)}\right]^{2\alpha_2(0)} .$$

In the following sections this expression is expanded in powers of $1/\sqrt{s}$ in the pionization region. We identify the zeroth order term with the scaling limit of the distribution and consider the first order term being proportional to $1/\sqrt{s}$ as the main part of the deviation from the scaling distribution.

3. The scaling term

For the derivation of the scaling term we express all kinematic variables in eq. (23) by c.m. energy E and c.m. momentum $(q_\parallel, q_\perp)$ of particle c in the leading order in $\sqrt{s}$. In the pionization region $q_\parallel, q_\perp$ remain finite at $s \to \infty$. We have

$$\begin{aligned} &M^2 \approx s, \\ &-u_1 \approx \sqrt{s}(E-q_\parallel), \qquad -u_2 \approx \sqrt{s}(E+q_\parallel), \\ &\tau_1 \approx \frac{E-q_\parallel}{\sqrt{s}}, \qquad \tau_2 \approx \frac{E+q_\parallel}{\sqrt{s}}, \\ &\frac{u_1 u_2}{M^2} \approx m^2 + q_\perp^2 = \mu^2, \\ &c-b \approx -\frac{\Omega_1\Omega_2}{\Omega_1+\Omega_2} q_\perp^2, \qquad c \approx \tfrac{1}{2}\, s\,(\Omega_1+\Omega_2), \\ &c-d_1 \approx \Omega_1\sqrt{s}(E+q_\parallel), \qquad c-d_2 \approx \Omega_2\sqrt{s}(E-q_\parallel), \\ &d_1+d_2 \approx (\Omega_1+\Omega_2)s . \end{aligned} \tag{24}$$

According to the assumed multiperipheral exponential behaviour in the momentum transfers t_1, t_2 the integrand of eq. (23) is strongly damped by the factor

$$\exp\,\{-(c-d_1)\,z-(c-d_2)x-(d_1+d_2)xz\} \quad \text{for large } s .$$

Therefore we can extend the integrations limits in eq. (23) to infinity. Substituting

$$z' = (c-d_1)z , \qquad x' = (c-d_2)x , \tag{25}$$

and using eq. (24) we can write the exponential term in eq. (23) in the s- independent form

$$\exp\left\{-\frac{\Omega_1\Omega_2}{\Omega_1+\Omega_2}q_\perp^2\right\}\exp\left\{-z'-x'-\frac{\Omega_1+\Omega_2}{\Omega_1\Omega_2\mu^2}x'z'\right\}.$$

We perform the same substitution in the rest of the integrand and expand it in powers of $1/\sqrt{s}$. We find the leading term

$$F(q,s)=g\frac{\pi}{8}\frac{s^{\frac{1}{2}\widetilde{\alpha}_1(0)+\frac{1}{2}\widetilde{\alpha}_2(0)-1}}{\mu^2}$$
$$\times\int_0^\infty\int_0^\infty \mathrm{d}x'\,\mathrm{d}z'\exp\left\{-\frac{\Omega_1\Omega_2}{\Omega_1+\Omega_2}q_\perp^2\right\} \tag{26}$$
$$\times\frac{(1+\Omega_1\mu^2/z')^{2\alpha_1(0)}(1+\Omega_2\mu^2/x')^{2\alpha_2(0)}}{r_1r_2(r_1+r_2)\,[r_1(E+q_\parallel)]^{\widetilde{\alpha}_1(0)}[\Omega_2(E-q_\parallel)]^{\widetilde{\alpha}_2(0)}}$$
$$\times\exp\left\{-z'-x'-\frac{\Omega_1+\Omega_2}{\Omega_1\Omega_2\mu^2}x'z'\right\}z'^{\widetilde{\alpha}_1(0)}x'^{\widetilde{\alpha}_2(0)}.$$

We notice that the factor

$$\exp\left\{-\frac{\Omega_1\Omega_2}{\Omega_1+\Omega_2}q_\perp^2\right\}$$

provides a sharp dropp off in $q_\perp^2$:

$$F(q,s)\propto\exp\left\{-\frac{\overline{\Omega}_1\overline{\Omega}_2}{\overline{\Omega}_1+\overline{\Omega}_2}q_\perp^2\right\}. \tag{27}$$

$\overline{\Omega}_i$ denotes the effective values of Ω_i from eq. (14). In the case of vanishing slope of trajectories $\alpha'_i=0$ one has

$$\Omega_i=\overline{\Omega}_i=\alpha_i. \tag{28}$$

Setting $\widetilde{\alpha}_1(0)=\widetilde{\alpha}_2(0)=1$ we find that the resulting inclusive cross section (26) is independent of s and q in this approximation which gives the limiting scaling distribution.

For $\widetilde{\alpha}_1(0)=1$, $\widetilde{\alpha}_2(0)=\frac{1}{2}$ and *vice versa* $F(q,s)$ behaves as $F\propto s^{-\frac{1}{4}}$; for $\widetilde{\alpha}_1(0)=\widetilde{\alpha}_2(0)=\frac{1}{2}$ as $F\propto s^{-\frac{1}{2}}$. These expressions correspond to the dynamically non-scaling contributions in the language of Mueller-Regge-phenomenology [3, 14]. In the following we assume the intercepts of the output poles to be $\widetilde{\alpha}_1(0)=\widetilde{\alpha}_2(0)=1$ and treat the kinematical deviations from scaling.

4. Kinematically non-scaling contributions

To obtain kinematically non-scaling terms we take into consideration one term more than in eq. (24) at the expansion of the kinematical terms of eq. (23). The expansion of the structure function of eq. (23) leads to

$$F = c_1 + c_2/\sqrt{s} + \dots \; .$$

The expression

$$\frac{\Delta F}{F_{\mathrm{sc}}} = \frac{c_2}{c_1\sqrt{s}}$$

is the deviation from the scaling limit which we sought for. To demonstrate this we consider the approach to scaling in a special case.

4.1. Special case

Assuming $\tilde{\alpha}_1(0) = \tilde{\alpha}_2(0) = 1$ we set

$$q_{\parallel} = 0, \quad q_{\perp} = 0, \qquad m = 0,$$
$$m_{\mathrm{a}} = m_{\mathrm{b}}, \quad \Omega_1 = \Omega_2 = \Omega, \quad \alpha_1(0) = \alpha_2(0) = 0. \tag{29}$$

Most of the kinematical variables of eq. (23) can be calculated exactly. From eqs. (10), (16) and (22) it follows

$$u_1 = u_2 = m_{\mathrm{a}}^2, \; M^2 = s, \quad \tau_1 = \tau_2 = -m_{\mathrm{a}}^2/s,$$
$$b = \Omega s - 2\Omega\, m_{\mathrm{a}}^2, \qquad d_1 = d_2 = \Omega s,$$
$$c^2 = \Omega^2(s^2 - 4m_{\mathrm{a}}^2\, s), \tag{30}$$
$$c = \Omega s - 2\Omega m_{\mathrm{a}}^2 + \mathrm{O}\,(1/s),$$
$$(c-d_1) = (c-d_2) = -2\Omega m_{\mathrm{a}}^2 + \mathrm{O}\,(1/s),$$
$$(c-b) = 0 + \mathrm{O}(1/s), \qquad (d_1 + d_2) = 2\Omega s.$$

Substituting

$$x' = \sqrt{d_1 + d_2}\, x = \sqrt{2\Omega s}\, x,$$
$$z' = \sqrt{d_1 + d_2}\, z = \sqrt{2\Omega s'}\, z, \tag{31}$$

and using eqs. (29), (30) we get from eq. (23)

$$F(0,s) = g\frac{\pi}{32}s\int\limits_0^\infty\int\limits_0^\infty \frac{dx'\,dz'}{\Omega^2}e^{\sqrt{2\Omega}\,m_a^2(x'+z')} \tag{32}$$

$$\times\, e^{-x'z'}\,xz(1-x)(1-z)(1-x-z).$$

We replace x,z in the integrand by x', z' according to eq. (31) and expand the integrand in powers of $1/\sqrt{s}$.

$$F(0,s) = g\frac{\pi}{64}\int\limits_0^\infty\int\limits_0^\infty \frac{dx'dz'}{\Omega^3}\left[x'z' - \frac{2\sqrt{2}x'^2z'}{\sqrt{s}}\left(\frac{1}{\sqrt{\Omega}} - \sqrt{\Omega}\,m_a^2\right)\right]. \tag{33}$$

With an effective value $\bar{\Omega}$ for Ω it follows for $\Delta F/F_{sc}$,

$$\frac{\Delta F}{F_{sc}} \propto -\frac{1}{\sqrt{s}}\left(\frac{1}{\sqrt{\bar{\Omega}}} - \sqrt{\bar{\Omega}}m_a^2\right). \tag{34}$$

For $\bar{\Omega} < 1/m_a^2$ we find negative deviations from scaling. For intercepts $\alpha_i(0) > (0)$ the upper limit for $\bar{\Omega}$ to produce negative deviations from scaling is shifted to the right with increasing $\alpha_i(0)$.

4.2. General case

We choose $\tilde{\alpha}_1(0) = \tilde{\alpha}_2(0) = 1$ to have scaling in the zeroth order and set $q_\parallel = 0$. Then we carry out a procedure similar to sect. 4.1. and obtain the following expression for not too large transverse mass μ ($\mu < m_a$):

$$\frac{\Delta F}{F_{sc}} = -\frac{b_1 + (3+b_2)\mu}{\sqrt{s}}. \tag{35}$$

Typical values of the parameters b_1 and b_2 are $b_1 \approx 1 \ldots 3$ GeV and $b_2 \approx -1 \ldots 0$ (see sect. 5). The functions b_1 and b_2 strongly depend on the parameters a_1 and a_2 of the residue function; b_1 is an increasing function with a_i, b_2 is a decreasing function with a_i. Eq. (35) implies that for fixed $q_\perp$ the deviation from scaling increases with growing mass of the observed particle. This agrees with the results of the thermodynamical model [1]. Humble [4] has proposed a model for inclusive distributions near $x = 0$ in terms of a product of triple Regge couplings. In agreement with data for $pp \to \pi^-$ + anything, $pp \to K^-$ + anything and $pp \to \bar{p}$ + anything at 1500 GeV/c and 24 GeV/c for fixed transverse momentum $q_\perp = 0.4$ GeV/c it is shown that the deviation from scaling depends on the mass of the observed particle.

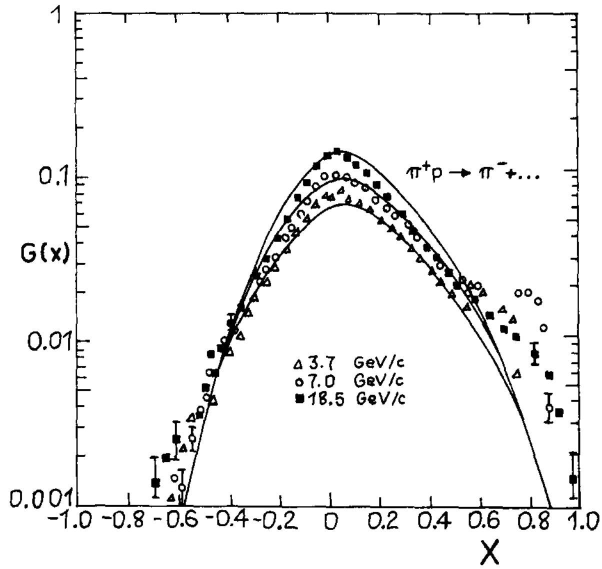

Fig. 2. The distribution

$$G(x) = \int \frac{1}{\sigma_T} E \frac{d\sigma}{dq_\parallel dq_\perp^2} dq_\perp^2$$

for the reaction $\pi^+ p \to \pi^-$ + anything at 3.7, 7.0 and 18.5 GeV/c computed from the multi-Regge model in comparison with the data of Alston - Garnjost et al. [28].

5. Comparison with experimental data

For a comparison with experimental data we perform computer calculations. Starting from eqs. (13) and (15) with the kinematical region given by eq. (17) we numerically integrate over this region. The resulting distribution

$$G(x) = \int \frac{1}{\sigma_T} E \frac{d\sigma}{dq_\parallel \, dq_\perp^2} \, d\, q_\perp^2$$

for the reaction $\pi^+ p \to \pi^-$ + anything at primary momenta 3.7, 7.0, 18.5 GeV/c is given in fig. 2 in comparison with experimental data [28]. For $\widetilde{\alpha}_1$ and $\widetilde{\alpha}_2$ we have assumed the pomeron ($\widetilde{\alpha}_1(0) = \widetilde{\alpha}_2(0) = 1$), for α_1 and α_2 we have chosen M-trajectories with $\alpha_i(t_i) = 0.5 + 0.85\, t_i$. The residue parameters a_i are fitted by $a_1 = 3$ (GeV/c)$^{-2}$ and $a_2 = 1.2$ (GeV/c)$^{-2}$. For this reaction the pionization contribution (eq. (1)) describes the data [28] well in the interval $-0.5 \leqslant x \leqslant 0.5$. Especially the approach to the scaling limit from below is in quantitative agreement with the data. Furthermore we notice that the asymmetry of the x-distribution is reproduced by our calculations, especially the maximum at $x = 0.05$. This fact arises not only from the mass asymmetry of the initial state but also from the difference of the parameters a_1 and a_2.

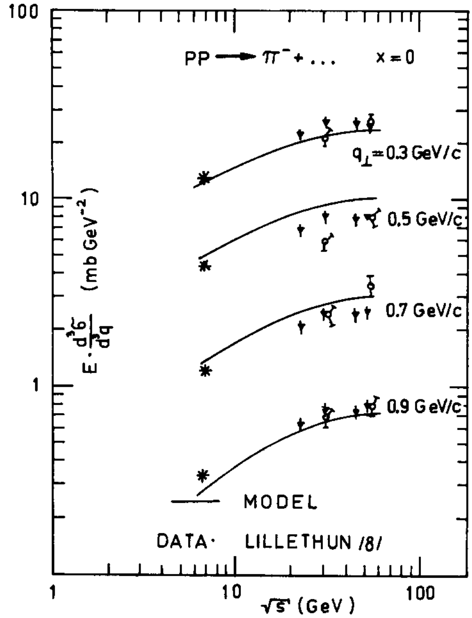

Fig. 3. The invariant production cross section $E\ \mathrm{d}^3\sigma/\mathrm{d}q$ for the reaction $\mathrm{pp} \to \pi^- +$ anything at $x = 0$ as function of the c.m. energy $\sqrt{s}$, computed from the multi-Regge model in comparison with the data reviewed by Lillethun [8]. The cross sections are given for transverse momenta $q_\perp = 0.3$, 0.5, 0.7 and 0.9 GeV/c. The data are taken from the work of the Saclay-Strasbourg group [29] (∇), the British-Scandinavian ISR Collaboration [30] (o), and Mück et al. [31] (*).

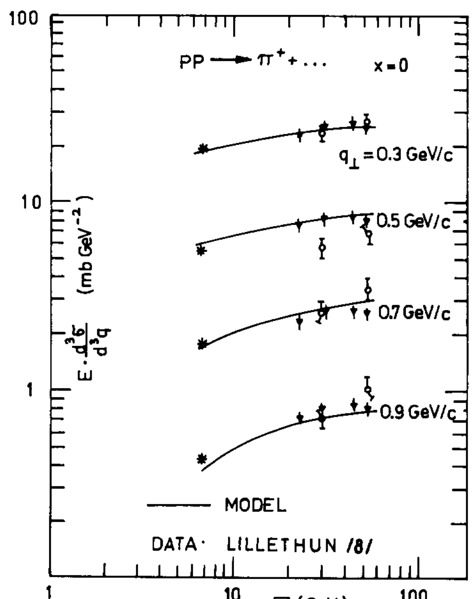

Fig. 4. The invariant production cross section $E\ \mathrm{d}^3\sigma/\mathrm{d}^3q$ for the reaction $\mathrm{pp} \to \pi^+ +$ anything at $x = 0$ as function of the c.m. energy $\sqrt{s}$, computed from the multi-Regge model in comparison with the data reviewed by Lillethun [8]. The cross sections are given for transverse momenta $q_\perp = 0.3$, 0.5, 0.7 and 0.9 GeV/c. The symbols are defined in fig. caption 3.

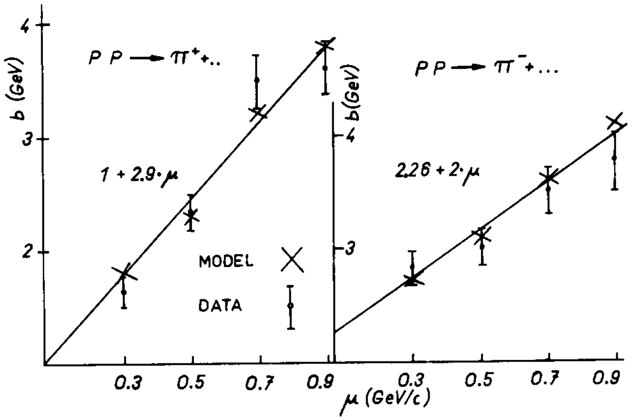

Fig. 5. Theoretical results from the multi-Regge model and experimental data [8] for the parameter b defined by:

$$\frac{F-F_{sc}}{F_{sc}} = \frac{-b}{\sqrt{s}}, \qquad F = E\,\frac{d^3\sigma}{d^3q},$$

for the reactions $pp \to \pi^+$ + anything and $pp \to \pi^-$ + anything. The function b is plotted over the transverse mass μ of the observed pions.

Table 1
Choice of the Regge parameters for the calculation of the invariant distribution in the central region according to the multi-Regge model

	$pp \to \pi^- + \ldots$	$pp \to \pi^+ + \ldots$
a_1	3.0	1.0
a_2	3.0	1.0
$\alpha_1(t_1)$	$0.5 + 0.85\,t_1$	$0.5 + t_1$
$\alpha_2(t_2)$	$0.5 + 0.85\,t_2$	$0.5 + t_2$
$\tilde{\alpha}_1(0)$	1.0	1.0
$\tilde{\alpha}_2\ (0)$	1.0	1.0

The chosen parameters for the reactions $pp \to \pi^+$ + anything and $pp \to \pi^-$ + anything are given in table 1. Fig. 3 and fig. 4 compare our numerical calculations for this reactions at $x = 0$ with experimental data [8] at primary momenta from 22 to 1500 GeV/c for various $q_\perp$. From these data and numerical calculations, respectively, we have determined the parameter b for the deviation from scaling

$$\frac{\Delta F}{F_{sc}} = -\frac{b}{\sqrt{s}}\,. \tag{36}$$

In fig. 5 the μ-dependence of this parameter is plotted. In agreement with the analytical results of sect. 4.2 b is a linear function of μ. With the parametrization of eq. (35) we obtain $b_1 = 1$ GeV, $b_2 = -0.1$ for π^+ and $b_1 = 2.26$ GeV, $b_2 = -1$ for π^-. In our model the stronger deviations from the scaling limit for the reaction pp $\to$ π^- + anything compared with pp $\to$ π^+ + anything for not too large $q_\perp$ are due to the larger value of the residue parameters a_i. The smaller value of b_2 is also explained by this choice of the residue parameters.

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References

[1] J. Ranft, Phys. Letters 41B (1972) 613.
[2] Htun Than and J. Ranft, Nuovo Cimento Letters 5 (1972) 655.
[3] Chan Hong-Mo, H.I. Miettinen, D.P. Roy and P. Hoyer, Phys. Letters 40B (1972) 406.
[4] S. Humble, Phys. Letters 40B (1972) 373.
[5] T. Ferbel, Phys. Rev. Letters 29 (1972) 448.
[6] R.P. Feynman, Phys. Rev. Letters 23 (1969) 1415.
[7] H. Meyer and W. Struczinski, DESY preprint DESY 72/40 (1972).
[8] E. Lillethun, A review of data on the production of charged particles at ISR, given at the 16th Conf. on high energy physics, Batavia, Illinois, Sept. 6–13, 1972.
[9] M. G. Albrow et al., Phys. Letters 40B (1972) 136,
A. Bertin et al., Phys. Letters 38B (1972) 260;
See also refs. [1, 4].
[10] R. Hagedorn, Nuovo Cimento Suppl. 3 (1965) 147;
R. Hagedorn and J. Ranft, Nuovo Cimento Suppl. 6 (1968) 169.
[11] R. Hagedorn and J. Ranft, Nucl. Phys. B48 (1972) 157.
[12] D. Amati, L. Caneschi and M. Testa, Phys. Letters 43B (1973) 186.
[13] D. Amati, A. Stanghellini and S. Fubini, Nuovo Cimento 26 (1962) 896.
[14] A.H. Mueller, Phys. Rev. D2 (1970) 2963.
[15] G.F. Chew and A. Pignotti, Phys. Rev. Letters 20 (1968) 1078.
[16] F. Zachariasen and G. Zweig, Phys. Rev. 160 (1967) 1326.
[17] C.E. DeTar, Phys. Rev. D3 (1971) 128.
[18] A. Pignotti and P. 'Ripa, Phys. Rev. Letters 27 (1971) 1538.
[19] Chung-I Tan, Report No COO-3130 TA-282 (1973).
[20] G.F. Chew, M.L. Goldberger and F.E. Low, Phys. Rev. Letters 22 (1969) 208.
[21] L. Caneschi and A. Pignotti, Phys. Rev. Letters 22 (1969) 1219;
L. Caneschi and A. Pignotti, Phys. Rev. 180 (1969) 1525.
[22] D. Silverman, Chung-I Tan, Nuovo Cimento 2A (1971) 489.
[23] L. Caneschi, Proc. of the 3rd Int. Colloquium on multiparticle reactions, Zakopane, 1972, p. 658–667.
[24] A. Bassetto and M. Toller, Nuovo Cimento Letters 2 (1971) 409;
A. Bassetto and M. Toller, Ann. Inst. Henri Poincaré, 18 (1973) 1;
A. Bassetto, KMU-preprint, Leipzig, KMU-HEP-7306, to appear in Fortsch. der Phys.

[25] L. Caneschi, Ref. TH 1704-CERN, submitted to Nucl. Phys.
[26] Chan Hong-Mo, K. Kajantie and G. Ranft, Nuovo Cimento 49 (1967) 157.
[27] R. Blutner and G. Ranft, Nucl. Phys. 47B (1972) 240.
[28] M. Alston-Garnjost et al., Phys. Letters 39B (1972) 402.
[29] M. Banner et al., paper 478 CERN (1972).
[30] B. Alper et al., paper 900, CERN (1972).
[31] H.J. Mück et al., Phys. Letters 39B (1972) 303.