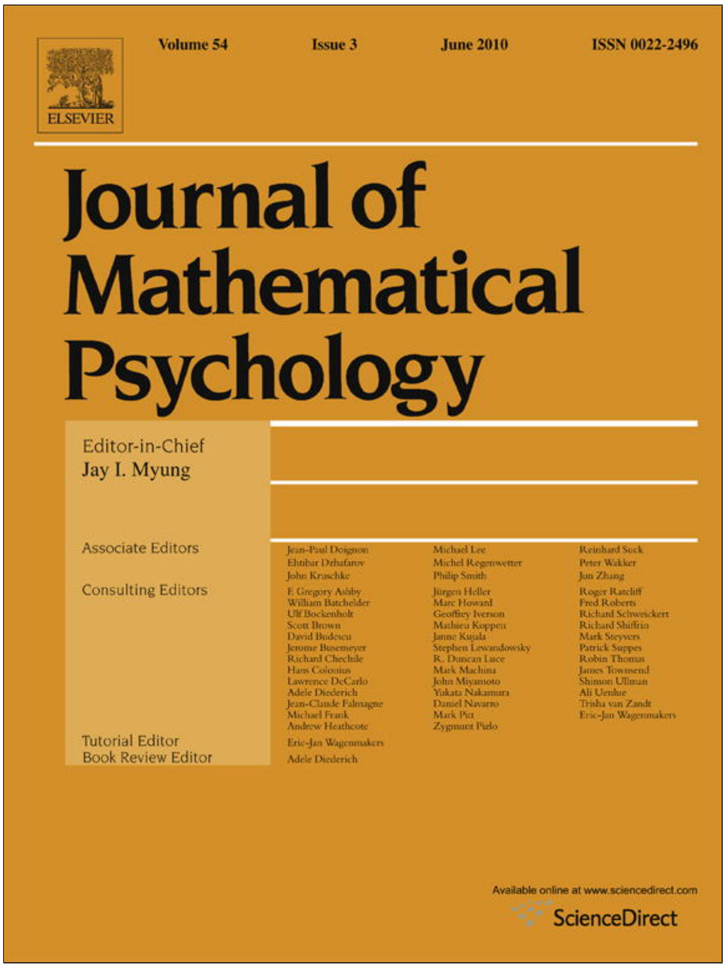
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A proposed test of temporal nonlocality in bistable perception

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ABSTRACT

The concept of temporal nonlocality is used to refer to states of a (classical) system that are not sharply localized in time but extend over a time interval of non-zero duration. We investigate the question whether, and how, such a temporal nonlocality can be tested in mental processes. For this purpose we exploit the empirically supported Necker–Zeno model for bistable perception, which uses formal elements of quantum theory but does not refer to anything like quantum physics of the brain. We derive so-called temporal Bell inequalities and demonstrate how they can be violated in this model. We propose an experimental realization of such a violation and discuss some of its consequences for our understanding of mental processes.

1. Introduction

The behavior of any system – no matter whether physical or mental – is generically described in terms of the evolution of its state (or its associated properties) as a function of time. Such a description is typically based on the assumption that the state of the system is precisely specified by a set of parameters fixing any possible measurable property of the system. However, it is known that this assumption is not always justified. In particular, in quantum mechanics the superposition principle implies the existence of states which do not have precisely specified features with respect to all properties. In other words, superposition states entail quantum nonlocality.

The fundamental idea to test such non-classical behavior in quantum physical situations is due to John Bell (Bell, 1966) who derived what are now known as Bell inequalities. Whenever Bell inequalities are violated, this is a key indication for non-classical behavior typical for quantum systems. In this way Bell inequalities have turned out to play a fundamental role in the interpretation of quantum theory. A temporal variant of them was proposed by Leggett and Garg in the mid 1980s (Leggett & Garg, 1985), again applied to quantum systems. The violation of temporal Bell inequalitites, not experimentally observed so far, would imply that events cannot be uniquely fixed in time. This is sometimes referred to as "nonlocality in time" (Mahler, 1997) or "temporal nonlocality" (Atmanspacher & Amann, 1998).

This general and very basic feature has inspired scientists to speculate whether non-classical behavior might also contribute to our understanding of puzzles outside the quantum domain, and maybe even outside the domain of physics. Actually, Bohr insisted since the 1920s, when he imported the concept of complementarity from psychology into quantum physics, that its significance extends over all fields of human knowledge, even into philosophical topics (Favrholdt, 1999; Holton, 1970). However, Bohr himself did not work out any concrete example in detail, and this has been the state of affairs for quite a while.

Since the 1970s some attempts can be witnessed to stretch the idea of quantum-like behavior in terms of time operators in stochastic systems (Gustafson & Misra, 1976; Tjøstheim, 1976) and of entropy production or information flow in dynamical systems (Atmanspacher & Scheingraber, 1987; Misra, 1978). Although stochastic and dynamical systems are clearly not quantum systems in the conventional sense, it became evident that particular features of the formal treatment of quantum systems can be properly applied to classical systems as well.

However, it was not before the 1990s when Bohr's original intuition of non-classical features even far beyond physics started to become investigated for concrete empirically accessible situations. To our knowledge, first pioneering work in this direction was carried out by Aerts and his group in Brussels (Aerts & Aerts, 1994), from which a long record of publications emerged that has recently been reviewed by Aerts (2009). Aerts and collaborators studied various kinds of problems in psychology and cognitive science, mainly from the viewpoint of quantum logic and quantum probabilities. A focus of applications of their work has been the formation and processing of concepts. Aspects of game theory (e.g., Eisert, Wilkens, & Lewenstein, 1999), context effects

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(e.g., Bruza & Cole, 2005), and decision making (e.g., Busemeyer, Wang, & Townsend, 2006) were later elaborated in detail by other groups.

Beginning in 2000, we developed an alternative approach, mainly embedded in the formal framework of algebraic quantum theory (Atmanspacher, Römer, & Walach, 2002). It was first referred to as "weak quantum theory", but later this was replaced by "generalized quantum theory" (GQT).[1] Different from the approach by Aerts, it is explicitly based on the non-commutative structure of the available set (algebra) of properties (observables). A key project demonstrating the viability of GQT refers to the bistable perception of ambiguous stimuli (Atmanspacher, Bach, Filk, Kornmeier, & Römer, 2008; Atmanspacher, Filk, & Römer, 2004, 2008), other applications have been indicated by Atmanspacher, Filk, and Römer (2006).

Remarkably, the various studies mentioned so far refrained from claiming premature relations to brain activity and aimed at genuinely psychological and cognitive descriptions of genuinely psychological and cognitive phenomena. We advocate the discussion of phenomena at the level at which their occurrence is observed since this avoids all kinds of unclear assumptions about interlevel relations. Of course, it is interesting to talk about neural correlates of cognition or consciousness as well, but this may not be the best choice to begin with.

In this spirit, our work, and that of other literature mentioned so far, is delineated from a number of quite popular proposals to address mind-brain issues in terms of quantum physics proper. The main representatives of such proposals (Wigner–Stapp, Umezawa–Vitiello, Beck–Eccles, Penrose–Hameroff) were reviewed by Atmanspacher (2004), and we do not discuss them here. A common ground of all of them is that, in one way or another, they try to invoke quantum physical brain mechanisms to describe or explain mental states and processes.

In Section 2 we introduce the formal framework of GQT and argue in favor of intralevel descriptions without interlevel assumptions. Then, in Section 3, we sketch a GQT-based model for bistable perception, the Necker–Zeno model, and show how it accounts properly for a number of empirical results. In Section 4 we introduce the idea of Bell inequalities and present a simple derivation of a temporal version of them. Section 5 shows how a temporal Bell inequality can be violated in the Necker–Zeno model, and Section 6 discusses how this can be interpreted. Section 7 summarizes our arguments and results.

2. Generalized quantum theory for cognitive systems

In 2002 it was proposed (Atmanspacher et al., 2002) to generalize quantum theory in such a way that some of its formal core features can be used for the description of systems outside quantum physics and even outside physics. The resulting generalized quantum theory (GQT) is a formal framework that specifies a set of mathematical conditions providing a mathematical description of any system which matches these conditions. In this sense, GQT is a general systems theory, and it is important to emphasize that it has (usually) nothing to do with a detailed quantum mechanical treatment of the system considered.

The elementary objects of GQT are states and observables. Observables can act on states and thereby not only yield results but also change states. Successive observations are related to a product of observables. This product on the set of observables leads to the concept of compatibility for commuting observables and incompatibility (or complementarity, respectively) for non-commuting observables (see Atmanspacher et al., 2006). Under general assumptions this leads to the concept of entanglement. The complementarity of observables is an essential prerequisite for observations which violate classical concepts of reality and causation.

[1] One equally supportive and critical remark concerning the notion of weak quantum theory was expressed by Marlan Scully at a conference in 2005: "Why do you call it weak if it is so strong?"

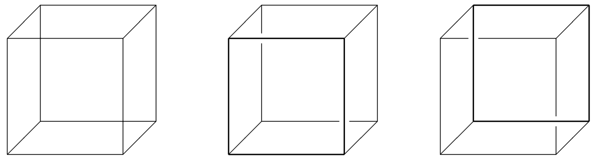

Fig. 1. The Necker cube (left) and the two perspectives under which it can be perceived (right).

This minimal set of conditions of GQT does not include dynamical laws, specific observables, invariances, decomposition rules, and other ingredients that are needed for a detailed description. These ingredients depend on the system considered and must be deliberately chosen. Since there is no universal language for descriptions of all systems, the corresponding choices are guided by a "descriptive level" which seems to be best suited for the system considered.

Descriptive levels can refer to various areas of physics (high-energy physics, statistical physics, thermodynamics, etc.), of chemistry (physical chemistry, organic chemistry, etc.), biology (biochemistry, genetics, etc.), of psychology (cognitive psychology, social psychology, etc.), of sociology, cultural studies and so on. It is obvious that there is no universal approach on which all these areas of science are founded. Yet it is interesting, and often useful, to look for relations between different descriptive levels, so-called interlevel relations (see, e.g. Bishop & Atmanspacher, 2006).

GQT offers an approach that can be applied to the descriptive level specifically relevant for cognitive processes. Using formal tools of quantum theory, the framework of GQT can be refined as to describe specific cognitive systems (rather than physical systems) in considerable detail. A number of interesting examples for corresponding applications have been worked out so far, some of which are reviewed by Atmanspacher et al. (2006). A particularly successful one among them addresses the perception of bistable stimuli, briefly bistable perception. It has been developed so far that experimental observations could be compared with predictions of a concrete model called the Necker–Zeno model (Atmanspacher et al., 2008).

3. The Necker–Zeno model

The Necker–Zeno model is a mathematical model describing the effective dynamics of switching mental states during the perception of ambiguous figures like the Necker cube (Fig. 1, left).

The Necker cube is a two-dimensional projection of a cube which does not fix the perspective under which it can be perceived (Necker, 1832). Two possible three-dimensional perspectives are consistent with the Necker cube, and usually one of them is perceived at a time (Fig. 1, right). Under continuing observation of an ambiguous figure the perceived perspective switches *spontaneously*, in an unforced fashion. For reviews see Blake and Logothetis (2002) and Long and Toppino (2004).

In experiments under controlled conditions, human subjects are requested to focus at a fixation cross in the center of the cube and indicate (e.g., by pressing a button) when a perspective switch occurred. This way, one obtains a sequence of switches as sketched in Fig. 2. From these data one can determine a distribution of dwell times T (or reversal rates $1/T$) for a perceptual state together with its moments, and one can determine the (cumulative) probabilities for a switch to occur. The observed distributions of T typically resemble a gamma distribution (see Brascamp, van Ee, Pestman, & van den Berg, 2005) with a power-law increase for small T and an exponential decrease for large T.

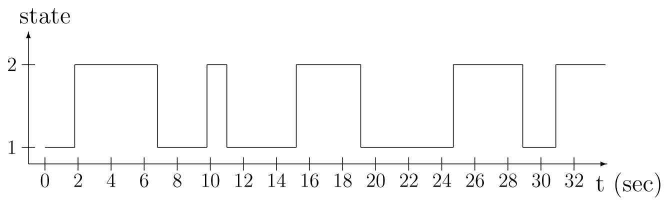

Fig. 2. Schematic representation of the bistable switching between two states 1 and 2 as a function of time t.

3.1. Mathematical formulation

The mathematical framework of the Necker–Zeno model is inspired by the mathematics of a two-state quantum Zeno effect (Misra & Sudarshan, 1977). However, we emphasize that we do not consider a phenomenon in the sense of quantum physics to be responsible for the switching dynamics. Our formal description exclusively refers to mental states and mental observables and their dynamics. It does not even address potential candidates for underlying brain activity.[2]

The details of the Necker–Zeno model have been described elsewhere (Atmanspacher et al., 2008, 2004). Here we restrict ourselves to a brief review of those features relevant for the purpose of this paper: the violation of temporal Bell inequalities. We first summarize the ingredients of the quantum Zeno model and then transform the concepts to the Necker–Zeno model.

The quantum Zeno model describes a two-state system with two complementary processes:

(D1) a continuous "rotation" in a 2-dimensional state space generated by a Hamiltonian $H = g\sigma_1$, where

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and g is a coupling parameter determining the time scale of the rotation. Hence, the unitary operator of the free evolution of the system without external influence is represented by

$$U(t) = e^{iHt} = \begin{pmatrix} \cos gt & i \sin gt \\ i \sin gt & \cos gt \end{pmatrix}$$

and yields a simple description of spontaneous switches between the two states of the system.

(D2) a discontinuous "reduction" process onto one of the two "eigenstates" $|+\rangle = \binom{1}{0}$ or $|-\rangle = \binom{0}{1}$ of the matrix

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

as the result of an "observation". Thus, the two σ_3-eigenstates may be represented by the projections

$$P_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad P_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

The eigenstates of σ_3 are stable under the "observation" process, but they are unstable under the dynamics according to σ_1. The corresponding two processes in (D1) and (D2) are complementary because the two matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1}$$

do not commute:

$$\sigma_1\sigma_3 \neq \sigma_3\sigma_1. \tag{2}$$

Now we are interested in two different types of probabilities:

(P1) The conditional probability $w_1(t)$ that the system is measured in state $|+\rangle$ at time t under the condition that it was in state $|+\rangle$ at time $t = 0$ and no measurement has been performed in between. This probability is given by:

$$w_1(t) = |\langle +|U(t)|+\rangle|^2 = \cos^2(gt). \tag{3}$$

After $t_0 := 1/g$ the probability w_1 has decreased to approximately 1/3. In this sense, t_0 characterizes the "decay" time of a state without external influence.

(P2) The conditional probability $w_N(t)$ that the system is measured in state $|+\rangle$ at time t under the condition that it was in state $|+\rangle$ at time $t = 0$ *and* under the condition that in N time intervals ΔT further observations were performed by which the system was always found to be in state $|+\rangle$. This probability is given by:

$$w_N(t) = \left|\langle +|(P_+ U(\Delta T) P_+)^N|+\rangle\right|^2 = \left(\cos^2(g\Delta T)\right)^N \tag{4}$$

with $t = N \cdot \Delta T$. The condition $\Delta T \ll t_0$ leads to the approximation:

$$w_N(t) \approx \exp(-g^2 \Delta T^2 \cdot N) = \exp\left(-\frac{\Delta T}{t_0^2} t\right). \tag{5}$$

Eq. (5) provides a relation for the average time $\langle T\rangle$ for the "decay" of the system under repeated observations at intervals ΔT, expressed in terms of the characteristic time t_0 of the unobserved system:

$$\langle T\rangle = \frac{t_0^2}{\Delta T} \quad \text{or} \quad t_0 = \sqrt{\langle T\rangle \cdot \Delta T}. \tag{6}$$

This relation between time scales pertains to the quantum Zeno effect. The smaller the observation intervals ΔT, the larger the average time $\langle T\rangle$ for the system to undergo a transition from $|+\rangle$ to $|-\rangle$.

3.2. Application to bistable perception

Now we summarize the essential results of the Necker–Zeno model by associating *mental* states and observables to the mathematical structures of the (physical) quantum Zeno effect. First, we interpret the two dynamical processes of the quantum Zeno model in terms of cognitive processes:

(D1′) We assume a "decay" of a perceptual state if a subject does *not* observe the ambiguous stimulus, i.e. if the stimulus is turned off. The probability that the mental state still refers to the perspective perceived before the stimulus was turned off is (for small t) given by:

$$w_0(t) = 1 - \frac{t^2}{t_0^2} + O(t^4) + \cdots \tag{7}$$

This is the short-time approximation of Eq. (3). The characteristic time t_0 is of the order of 300 ms, in accordance with the electrophysiological P300 component in event-related potentials. (For more details of this interpretation see Atmanspacher et al., 2004.)

(D2′) We assume a cognitive "update" process which takes place whenever a subject actually watches the stimulus. The mental state with its perspective of the cube is not stationary during the observation period, but it is updated in short time intervals ΔT. This update interval ΔT can be associated with the time scale at which the sequence of successive

[2] This exemplifies our insistence in Section 2 that phenomena should primarily be discussed at the level of description at which they are established. The rationale behind this is that their reduction to lower levels may be problematic if lower levels provide necessary but not sufficient conditions for higher-level phenomena. Of course, it remains entirely legitimate and even useful to look for correlations between higher-level phenomena and lower-level descriptions.

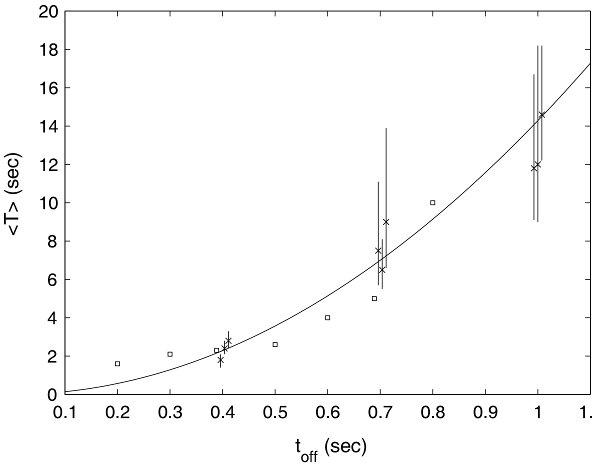

Fig. 3. Experimentally obtained mean dwell times $\langle T \rangle$ (inverse reversal rates) for the bistable perception of a discontinuously presented Necker cube. Crosses mark results from Kornmeier et al. (2007); for each off-time, $\langle T \rangle$ (including standard errors) is plotted for three on-times of 0.05 s, 0.1 s, and 0.4 s. Squares mark results from Orbach et al. (1966, no errors indicated) for an on-time of 0.3 s. The plotted curve shows $\langle T \rangle$ as a function of off-times t_{off} according to Eq. (6) with $\Delta T \approx 70$ ms and $t_{off} \approx t_0$.

stimuli can be cognitively resolved, $\Delta T \approx 30$ ms. (See again Atmanspacher et al., 2004, for further details.[3])

With this mental interpretation of the quantum Zeno effect in terms of the dynamics and time scales relevant for the perception of ambiguous stimuli, we can now interpret Eq. (6) with respect to bistable perception. Eq. (6) turns then into a relation between the average dwell time $\langle T \rangle$ of a perceived perspective (of the order of some seconds) and the time scales t_0 and ΔT as introduced above.

Relation (6) is clearly satisfied for these three cognitive time scales. Moreover, the predictive power of the model has been convincingly demonstrated with empirical results obtained under *discontinuous* stimulus presentation if it is possible to vary one of the time scales (t_0) as an independent variable and measure another one ($\langle T \rangle$) as a function of t_0. Assuming that ΔT remains constant, Eq. (6) predicts a quadratic dependence of $\langle T \rangle$ on t_0.

Under certain conditions, the time scale t_0 can be approximated by the off-time in discontinuous presentation, so it is indeed possible to test the model with experimental data. A comparison of observations by Kornmeier, Ehm, Bigalke, and Bach (2007) and Orbach, Zucker, and Olson (1966) with the predictions of the Necker–Zeno model is shown in Fig. 3. The plotted symbols show observed values of $\langle T \rangle$ as a function of off-times. In addition to and independent of the quadratic dependence that the model predicts Eq. (6), the best polynomial fit to the data (solid line) is also quadratic and yields $\Delta T \approx 70$ ms.

Note that the lowest off-time $t_{off} = 200$ ms in Fig. 3 has the largest relative deviation from the predicted curve. This is due to the fact that for off-times smaller than 300 ms, t_{off} can no longer be used to mimic t_0 but still has an influence on $\langle T \rangle$. In order to investigate this influence, we used data obtained from Kornmeier et al. (2007) for small off-times. The details of the analysis have been described by Atmanspacher et al. (2008), and Fig. 4 shows the results.

As can be seen in Fig. 4, the Necker–Zeno model describes a decrease of reversal rates, corresponding to an increase of $\langle T \rangle$, for decreasingly short off-times. This effect indicates opposing trends for long and short off-times, separated by a critical time scale of the order of 300 ms. To our knowledge there is no other model that predicts this non-trivial behavior of reversal rates (dwell times) correctly.

[3] Recent work by vanRullen, Reddy, and Koch (2005) suggests another empirical demonstration of ΔT of the same order of magnitude as an update interval: the "wagon wheel" illusion. There is evidence for an attention-dependence of ΔT in this scenario.

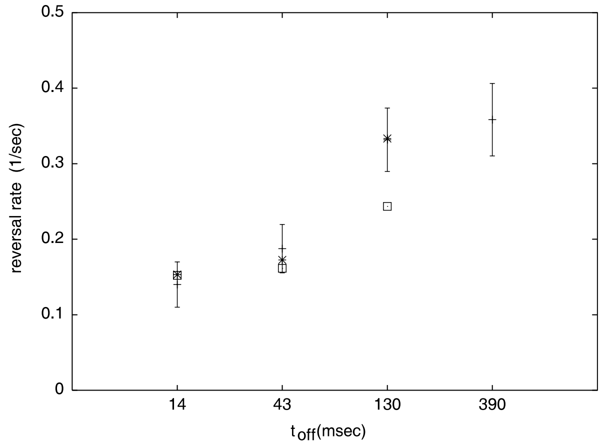

Fig. 4. Reversal rates versus small off-times t_{off}: (a) Experimental observations (crosses) from Kornmeier et al. (2007), with standard error of the mean; (b) best fit of reversal rates to experimental data according to the Necker–Zeno model (asterisks); (c) results (squares) for assumed parameters $\Delta T = 30$ ms and $t_0 = 300$ ms. The reversal rate for $t_{off} = 390$ ms is disregarded in the fit since it is outside the scope of small off-times.

To conclude this section with one more reference to empirical data, the Necker–Zeno model in its original form (Atmanspacher et al., 2004) gives an inappropriate description of the distribution of small dwell times. Introducing a natural refinement of the model, in which either t_0 or ΔT (or both) are time-dependent and serve to describe the initial phase of the process, we achieved a distribution (Atmanspacher et al., 2008) consistent with the observed gamma distribution of dwell times (see, e.g., Brascamp et al., 2005).

As shown by Atmanspacher et al. (2008), the resulting refined Necker–Zeno model describes an exponential decay for large dwell times T (as in Eq. (5)) and a power-law increase for small values of T (different from Eq. (5)). Since the predicted behavior for small values of T depends on the way in which the initial phase of the process is taken into account, there is the option to distinguish between the two. Some kind of attention relaxation could be a significant factor for a cognitive interpretation of the competing kinds of initial behavior. Indeed, vanRullen (2009), studying the paradigm of the wagon-wheel illusion, found that a corresponding updating interval ΔT can increase from 70 ms up to approximately 120 ms under split attention.

4. Temporal Bell inequalities

When Bell derived the famous Bell inequalities (Bell, 1966) they were – at least in part – conceived as an attempt to disprove quantum theory. Bell's inequalities rely on an assumption of objective realism and can be violated if reality behaves as predicted by quantum theory. When the first experimental data indeed violated the inequalities, Bell himself was among the first to accept that quantum theory provides a correct picture of reality. After the precise and convincing measurements of Aspect and collaborators in the early 1980s (Aspect, Dalibard, & Roger, 1982), only few mostly skeptical physicists maintained doubts.

In detail, Bell's inequalities essentially rely on three fundamental assumptions (compare, e.g. d'Espagnat, 1979).

1. Objective Realism: For any system and any possible measurement which can be performed on this system, it is assumed that the result which this measurement would yield has a fixed

albeit unknown value at any moment. (An "omniscient" being could predict which result any measurement on the system would yield.) In other words: The ontic state[4] of a system, unknown to us, carries the information about the sharp value resulting from any measurement which in principle could be performed on this system.

2. Locality (Separability): A measurement performed at a particular spatial location cannot have any influence whatsoever on the results of another measurement performed at a different location if the second measurement is carried out within the causal complement (as defined by the relativistic lightcone) of the first measurement.
3. Experimental Induction: Whenever particular measurements of systems repeatedly prepared in the same state yield particular results, these results characterize the system state even if the measurements are not actually performed.

The first assumption is violated by the experimental results of Aspect et al. (1982) and many other groups, which are correctly described by quantum theory. The other two assumptions are necessary in order to test the violation of Bell's inequalities in quantum systems. As most forms of Bell's inequalities make statements about the correlations of two measurements, it must be excluded that one of the measurements has an influence on the result of the other measurement. (This is the so-called "non-invasiveness" of a measurement.) In most cases one performs measurements on a pair of entangled particles in an Einstein–Podolsky–Rosen (EPR)- or Bell-state. Locality is needed in order to guarantee that these two measurements do not influence each other.

The third assumption (induction) is needed to ensure that the result of one measurement performed at one of the particles allows us to make a statement about the result of a hypothetical (but not actually performed) measurement on the other particle. (In an EPR-state, the results of two measurements of the same observable on the two particles usually would be anticorrelated. This is assumed to hold true even if different measurements are performed.)

In all cases with violations of Bell's inequalities, the measurements performed at the two particles refer to non-commuting observables, e.g. spins in different directions. It can be shown that the correlation functions for measurements, which occur in Bell's inequalities, will always satisfy Bell's inequalities as long as all measured observables commute.

Even though the Necker–Zeno model involves the non-commuting operators σ_1 and σ_3, a well-defined measurement prescription only exists for σ_3. For this reason, it is not possible to test a violation of the standard Bell inequalities in an experiment which uses only this observable. Furthermore, Bell's inequalities are usually tested for entangled quantum systems, and at present we have no idea of how to prepare entangled cognitive states in the usual sense of entanglement. However, there is a viable alternative: temporal nonlocality (see Atmanspacher & Filk, 2003).

In 1985, Leggett and Garg (1985), see also Mahler (1994), derived a *temporal* version of Bell's inequalities which involves only one observable. The underlying idea is the following: if an observable (σ_3 in our case) does not commute with the dynamics, i.e. the Hamiltonian of the system (σ_1 in our case), then two measurements of the same observable performed at different times do, in general, not commute.

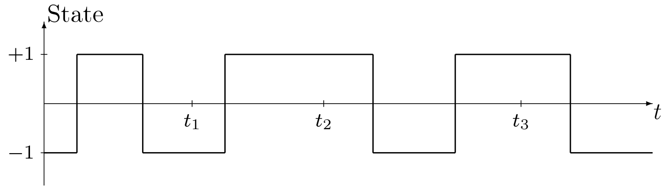

Fig. 5. A classical trajectory assumes at each moment in time a definite state (here one of two possible states). With respect to three instances t_1, t_2, and t_3 it falls into one of $2^3 = 8$ possible classes (cf. Table 1, left). For the shown history the states are $(-1, +1, +1)$.

Table 1
A classical trajectory for a two-state system falls into one of eight possible classes with respect to the states assumed at three different moments of time.

$s(t_1)$	$s(t_2)$	$s(t_3)$	$N^-(t_1,t_3)$	$N^-(t_1,t_2)$	$N^-(t_2,t_3)$
+1	+1	+1			
+1	+1	−1	×		×
+1	−1	+1		×	×
+1	−1	−1	×	×	
−1	+1	+1	×	×	
−1	+1	−1		×	×
−1	−1	+1	×		×
−1	−1	−1			

More technically speaking, if A corresponds to a measurement of an observable at time $t = 0$, then $A(t) = U(t)AU^{-1}(t)$ corresponds to the measurement of the same observable at time t, where $U(t) = \exp(\mathrm{i}Ht)$ is the time evolution operator. If A does not commute with $U(t)$ and, therefore, not with H, then A does not commute with $A(t)$. This makes it possible to formulate temporal Bell inequalities with only one observable.

We are now going to derive a particular form of temporal Bell inequality inspired by an argument of Kochen and Specker (d'Espagnat, 1979; Kochen & Specker, 1967; Mermin, 1990), before we show how they are violated in the Necker–Zeno model (Section 5) and discuss possible interpretations of such a violation (Section 6). As in the Necker–Zeno model of Section 3, we refer to a simple two-state system, i.e., a system which can only assume two different states. (Generalizations to systems with an arbitrary number of states are possible.) The assumption of "reality" (as described above) implies that at each moment t the system is in one of the two states. The "history" of the system is then given by a classical trajectory which, at each moment, assumes one of two possible values.

If we now specify three different moments t_1, t_2 and t_3 (as in Fig. 5) and define $s(t_1)$ to be the state of the system at time t_1 (and similarly $s(t_2)$ and $s(t_3)$), we can say that any classical trajectory falls into one of $2^3 = 8$ possible classes summarized in Table 1 (left).

Let us now consider an ensemble of classical trajectories and denote by $N^-(t_1, t_3)$ the number of cases in which the system is in *different states at* t_1 *and* t_3 (i.e., where the product of the state values is -1). Similarly, we define $N^-(t_1, t_2)$ and $N^-(t_2, t_3)$. From Table 1 (right) it is obvious that any of the four possibilities contained in $N^-(t_1, t_3)$ is also contained in either $N^-(t_1, t_2)$ or $N^-(t_2, t_3)$. Therefore, simple set-theoretical considerations provide the inequality:

$$N^-(t_1,t_3) \leq N^-(t_1,t_2) + N^-(t_2,t_3). \quad (8)$$

This is what we are looking for. The standard formulation of Bell's inequalities refers to two observations at two different (but entangled) particles and evaluates the correlations between the two particles. By contrast, the derived temporal version of

[4] Sometimes, in particular by proponents of the many-world interpretation of quantum mechanics (deWitt, 1970; Everett, 1957), objective realism is simply defined as the ontic existence of a quantum state. We interpret this ontic existence in the sense of "elements of reality" as discussed by Einstein, Podolsky, and Rosen (1935). See also d'Espagnat (1979) and Mermin (1990) for further discussion in the same spirit.

a Bell inequality refers to successive measurements of the same observable at different times.

Dividing Eq. (8) by the total number of trajectories we obtain an inequality for the corresponding probabilities:

$$p^-(t_1, t_3) \le p^-(t_1, t_2) + p^-(t_2, t_3). \quad (9)$$

Assuming time-translation invariance, i.e. these probabilities do not depend on absolute times but only on time differences, leads to:

$$p^-(t_3 - t_1) \le p^-(t_2 - t_1) + p^-(t_3 - t_2). \quad (10)$$

As we shall see in the next section, this inequality can be violated in the Necker–Zeno model.

5. Violating temporal Bell inequalities in the Necker–Zeno model

The Necker–Zeno model is based on a mathematical formalism which involves two complementary features: (i) the Hamiltonian generating the dynamics of unperturbed evolution and (ii) the observable leading to the "reduction" of the perceived stimulus into one of its two alternative perspectival representations. Since (i) and (ii) can be represented by non-commuting operators, the model suggests the possibility of violating the temporal version of Bell's inequalities.

According to Eq. (3) the conditional probability for a system to be in state $|+\rangle$ at time t_1 and to be measured in state $|+\rangle$ at time t_2 is given by

$$w_{++}(t_1, t_2) = \cos^2(g(t_2 - t_1)). \quad (11)$$

Therefore, the conditional probability of being in state $|+\rangle$ at time t_1 and being measured in state $|-\rangle$ at time t_2 is given by

$$w_{+-}(t_1, t_2) = \sin^2(g(t_2 - t_1)), \quad (12)$$

and for the reverse transition we obtain the same probability:

$$w_{-+}(t_1, t_2) = \sin^2(g(t_2 - t_1)). \quad (13)$$

In all these conditional probabilities the condition refers to the state at t_1. Furthermore, the probabilities involved in the temporal version of Bell's inequalities are probabilities $p^-(t_1, t_2)$ for anti-correlated observations, referring to the case that the perception at time t_1 differs from the perception at time t_2. In the Necker–Zeno model we obtain:

$$p^-(t_1, t_2) = \frac{1}{2}(w_{+-}(t_1, t_2) + w_{-+}(t_1, t_2)) = \sin^2 g(t_2 - t_1). \quad (14)$$

We now have to find differences between times t_1, t_2 and t_3 for which these probabilities violate inequality (10). We consider the special case for which $\tau := t_3 - t_2 = t_2 - t_1$, i.e., for which the time intervals on the right hand side of the inequality are equal. In this case we obtain

$$p^-(2\tau) \le 2p^-(\tau). \quad (15)$$

In the Necker–Zeno model this inequality is maximally violated if:

$$g\tau = \frac{\pi}{6}, \quad (16)$$

which yields

$$\sin^2(g \cdot 2\tau) = \frac{3}{4} \quad \text{and} \quad \sin^2(g\tau) = \frac{1}{4}. \quad (17)$$

For $t_0 = 1/g \approx 300$ ms we obtain

$$\tau = \frac{\pi}{6} \cdot t_0 \approx 157 \text{ ms}. \quad (18)$$

A possible experimental set-up to measure the violation of the temporal Bell's inequality could be as follows: An ambiguous (Necker cube) stimulus is presented up to time $t_1 = 0$ and then turned off. At this point, the subject has to remember the perspective perceived most recently. At time $t_2 \approx 157$ ms or $t_3 \approx 314$ ms the stimulus is turned on again, and the subject has now to indicate whether the perceived perspective has switched or not. The ratio of cases with switched perspective over the total number of cases provides the probabilities $p^-(\tau)$ and $p^-(2\tau)$ needed to check inequality (10).

So far, we took into account the full probability for a perspective switch in a time interval τ (Eq. (14)). We cannot expect this equation to hold for very long time intervals, in particular the periodicity of the probability may be an artifact of the extension of our model beyond plausible time scales.

In this context, we note that the approximation for the decay probability (Eq. (7)) can also be used to check violations of temporal Bell inequalities. The inequality (15) expresses a "sublinearity" for the probability $p^-(\tau)$ which is satisfied whenever $p^-(\tau)$ increases linearly or less than linearly as a function of τ. It is violated if $p^-(\tau)$ increases faster than linear. Eq. (7) implies

$$p^-(\tau) \approx g^2\tau^2 + O(\tau^4), \quad (19)$$

which indeed increases faster than linear and thus violates the temporal Bell inequalities.

The experimental scheme described above includes an issue that is problematic for almost all direct attempts to test temporal Bell inequalities: the difficulty of "non-invasive" measurements. A particular history of a system should belong to one of the eight classes described in Table 1, independent of whether or not an observation is made at some instant in time. Or, in other words, a measurement at time t_1 or t_2 should not change the class to which the history belongs as compared to the case of no measurement performed.

For ordinary (non-temporal) Bell inequalities, the non-invasiveness of the first measurement can be simply secured if (assuming locality) the second measurement is performed within the causal complement of the lightcone of the first. For temporal Bell inequalities this is impossible to do since both measurements are performed at the same local system. For this reason, one has to find a way to keep the degree of invasiveness due to the first measurement as small as possible. Ideally, this would be realized by an experiment distinguishing between the correlation or anti-correlation of the two perceptions involved rather than comparing the two individual perceptions.

However, there is an option for temporal Bell inequalities to permit an even more direct answer to the question of objective realism. In general, Bell's inequalities involve expectation values of products of non-commuting observables, $\langle A \cdot B\rangle$, and any measurement of A or B is "invasive" in the sense that it leads to a change of the state of the system. The usual trick to avoid this "invasiveness" (under the assumption of locality) is to replace the expectation value $\langle A \cdot B\rangle$ by $\langle A_1 \cdot B_2\rangle$, where the indices refer to different particles and, therefore, A_1 and B_2 commute. For certain entangled states of such two particles, the strict correlations allow us to induce the result of a hypothetical measurement of B_1 at particle 1 from the actually performed measurement of B_2 at particle 2 (via the induction hypothesis).

Temporal Bell inequalities involve expectation values of the type $\langle A(t_1) \cdot A(t_2)\rangle$. From the viewpoint of quantum mechanics we again have the problem that $A(t_1)$ and $A(t_2)$ do not commute and, therefore, it is not possible to perform non-invasive measurements of both observables. However, for cognitive systems it may be the case that, instead of measuring $A(t_1)$ and $A(t_2)$ separately (and, therefore, violating the non-invasiveness assumption), one can get the information about the product $A(t_1)A(t_2)$ in *one* measurement. For the case of bistable perception, one may ask subjects only to indicate whether or not the perception of the cube has changed from t_1 to t_2. This does not necessarily involve explicit knowledge of the two perspectives themselves. In particular, we suggest that potential memory and/or response effects due to an explicit measurement of $A(t_1)$ may be significantly reduced in an experimental arrangement that is sufficiently subtle to allow subjects to respond to the product $A(t_1)\,A(t_2)$ only after t_2.

In this context we should mention that measurements of the cumulative dwell time distribution (see, e.g., Fig. 3 in Brascamp et al., 2005) show a power-law behavior $p(T) \propto T^k$ with $k \gg 1$ for small values of T, which obviously does not satisfy Eq. (14). However, since the distribution is obtained from successive explicit responses to individual perceptions, this corresponds to temporally local measurements that are invasive. In other words, the explicit response to a perception distinguishes an instance $t = 0$, and this contradicts the assumption of time-translation invariance for the derivation of the temporal Bell inequality equation (10). Therefore, the power-law behavior of small dwell times does not indicate non-classical behavior of the temporal evolution of mental states.

6. Discussion

For the historical Bell inequalities and their prehistory, it was a key question which of the assumptions discussed in Section 4 should be relaxed or even abandoned. The famous articles by Einstein et al. (1935) and by Bell (1966) both revolved around this issue. For Einstein, who was convinced that local "elements of reality" exist, the conclusion from nonlocal correlations was that quantum theory is not complete. He argued that quantum theory provides only statistical information about a system, whose individual states will some day become describable by (so far "hidden") local variables.

Bell's inequalities are formulated in such a general way that they hold for any theory based on such hidden local variables (Bell, 1966). Their violation in quantum systems (Aspect et al., 1982) indicates that quantum physics is not based on hidden local variables. Einstein's "local elements of reality" are thereby refuted.

A most important consequence of this result is that nonlocal correlations between subsystems must not be understood in terms of causal interactions, possibly faster than light, between those subsystems. Einstein insisted to exclude such "spooky actions-at-a-distance" because, in his view, they would make any science impossible. Today we know that nonlocal correlations have nothing to do with "spookiness".

So, a proper understanding of holism and causation is the key to an appropriate interpretation of systems which violate Bell inequalities. This also applies to their temporal variant, though in a slightly different way. While the effects of nonlocality à la Bell and Aspect are mostly discussed in terms of spatial relations between spatial subsystems, temporal Bell inequalities refer to relations between temporal segments of the history of a system.

Violated temporal Bell inequalities entail that a state of a system is nonlocally distributed along the time axis. System states are improperly described if they are sharply localized at definite instances of time. They are "stretched" over an extended time interval that may depend on the specific system considered. Within this interval, relations such as "earlier" or "later" are illegitimate designators of the system state. This is just another way of saying that it is impossible to define causal relationships within such a time interval (Filk & von Müller, 2009).

In the Necker–Zeno model, this could be interpreted due to a "superposition" state, actualizing neither one nor the other perspective, but residing somehow "in between", offering the potential to actualize either one or the other perspectival state. Needless to say, this resembles the idea of a "reduction" of a quantum superposition state very closely. We should indicate that a similar scenario of mental temporal nonlocality was recently proposed as characteristic of processes of cognitive decision making (Busemeyer et al., 2006).

It is tempting to relate this temporal nonlocality to a "window of temporal nowness", a concept that transcends a sharp boundary of presence between past and future (Filk & von Müller, 2009; Pöppel, 1997). Some indications of what this may have to do with the Necker–Zeno model have been given by Franck and Atmanspacher (2008), but the idea itself is much older and dates back at least to James' notion of the "specious present", a present mental state extending over a time interval rather than fixed to an instant of vanishing duration. Acategorial states as discussed by Atmanspacher and Fach (2005) might be interesting candidates for temporal nonlocality as a mental feature.

An interesting remark by Sudarshan (1983), one of the discoverers of the quantum Zeno effect, illustrates how the phenomenal experience of a temporally holistic state could be imagined. Sudarshan speculates about a mode of awareness in which "sensations, feelings, and insights are not neatly categorized into chains of thoughts, nor is there a step-by-step development of a logical-legal argument-to-conclusion. Instead, patterns appear, interweave, coexist; and sequencing is made inoperative. Conclusion, premises, feelings, and insights coexist in a manner defying temporal order."

It is a necessary condition for temporally nonlocal correlations that the dynamics of the system considered is governed by operators that do not commute. This is the case in the Necker–Zeno model, so that bistable perception may indeed exhibit non-classical behavior although the corresponding cognitive mechanisms are not treated on the basis of a genuine quantum physical system. On a more general basis, we argued previously (Atmanspacher & Filk, 2003) that non-commuting time operators might be generic features of systems with temporal nonlocality. It is known that such time operators bear an intimate connection with both inconsistent histories and temporal nonlocality (Atmanspacher & Amann, 1998). A thematically related but more general approach to relate physical and mental time to one another was recently proposed (Primas, 2007, 2008).

7. Summary

In quantum physics, violations of Bell inequalities are regarded as the most profound proof of evidence for quantum entanglement and associated nonlocal correlations. Generalized quantum theory, a formal framework based on non-commuting observables, allows us to discuss quantum-like behavior also in non-quantum systems and even in non-physical systems. Various approaches have demonstrated this successfully for cognitive processes in mental systems, such as decision making, context effects, concept formation, and others.

One particular application, which we have worked out in detail, refers to the bistable perception of ambiguous stimuli. The corresponding Necker–Zeno model has been proposed and refined in recent years and is now supported by several pieces of experimental evidence. The dynamics of the ambiguous perceptual states (percepts) can be modeled by non-commuting operators and, thus, suggests non-classical effects.

One such effect is the potential existence of mental "superposition" states. In order to detect such states, it is desirable to have a Bell-type inequality tailored to mental systems, which can be violated if the system is in a superposition state. We derive a special variant of such a Bell inequality for temporal histories of systems rather than for decomposed subsystems.

This temporal Bell inequality, formulated in the spirit of the Kochen–Specker scheme, can indeed be violated in the Necker–Zeno model. We determine the parameters of maximal violation and propose an experimental scheme for tests. A major obstacle for conclusive results will be the inevitability of invasive measurements in bistable perception.

If mental superposition states can in fact be established by violations of temporal Bell inequalities, this has remarkable consequences. More than a century ago, William James already pointed out that the concept of a "stream of consciousness" entails the necessity of intermediate states in between distinct percepts. These states can easily be understood as unstable dynamical states. Superposition states according to the Necker–Zeno model would suggest a radically non-classical alternative: states extending over time with non-causal correlations within their window of nowness.

We emphasize that the Necker–Zeno model refers to cognitive activity independent of any possibly underlying brain mechanisms. Although neural correlates of cognitive processes are an interesting area of research in themselves, we argue that, to begin with, phenomena should be described at the descriptive level at which their occurrence is observed. This has the advantage that no unclear or obscure interlevel relations need to be assumed.

References

Aerts, D. (2009). Quantum structure in cognition. *Journal of Mathematical Psychology*, *53*, 314–348.

Aerts, D., & Aerts, S. (1994). Applications of quantum statistics in psychological studies of decision processes. *Foundations of Science*, *1*, 85–97.

Aspect, A., Dalibard, J., & Roger, G. (1982). Experimental test of Bell's inequalities using time-varying analyzers. *Physical Review Letters*, *49*, 1804–1807.

Atmanspacher, H. (2004). Quantum approaches to consciousness. In Edward N. Zalta (Ed.), *The Stanford encyclopedia of philosophy* (Winter 2004 Edition).

Atmanspacher, H., & Amann, A. (1998). Positive operator valued measures and projection valued measures of non-commutative time operators. *International Journal of Theoretical Physics*, *37*, 629–650.

Atmanspacher, H., Bach, M., Filk, T., Kornmeier, J., & Römer, H. (2008). Cognitive time scales in a Necker–Zeno model for bistable perception. *Open Cybernetics and Systemics Journal*, *2*, 234–251.

Atmanspacher, H., & Fach, W. (2005). Acategoriality as mental instability. *Journal of Mind and Behavior*, *26*, 181–206.

Atmanspacher, H., & Filk, T. (2003). Discrimination and sequentialization of events in perception. In R. Buccheri, et al., (Eds.), *The nature of time: Geometry, physics, and perception* (pp. 67–75). Dordrecht: Kluwer.

Atmanspacher, H., Filk, T., & Römer, H. (2004). Quantum Zeno features of bistable perception. *Biological Cybernetics*, *90*, 33–40.

Atmanspacher, H., Filk, T., & Römer, H. (2006). Weak quantum theory: Formal framework and selected applications. In G. Adenier, et al., (Eds.), *Quantum theory: Reconsideration of foundations – 3* (pp. 34–46). New York: American Institute of Physics.

Atmanspacher, H., Filk, T., & Römer, H. (2008). Complementarity in bistable perception. In H. Atmanspacher, & H. Primas (Eds.), *Recasting reality – Wolfgang Pauli's philosophical ideas in contemporary science* (pp. 135–150). Berlin: Springer.

Atmanspacher, H., Römer, H., & Walach, H. (2002). Weak quantum theory: Complementarity and entanglement in physics and beyond. *Foundations of Physics*, *32*, 379–406.

Atmanspacher, H., & Scheingraber, H. (1987). A fundamental link between system theory and statistical mechanics. *Foundations of Physics*, *17*, 939–963.

Bell, J. S. (1966). On the problem of hidden variables in quantum theory. *Reviews of Modern Physics*, *38*, 447–452.

Bishop, R. C., & Atmanspacher, H. (2006). Contextual emergence in the description of properties. *Foundations of Physics*, *36*, 1753–1777.

Blake, R., & Logothetis, N. (2002). Visual competition. *Nature Neuroscience*, *3*, 1–11.

Brascamp, J. W., van Ee, R., Pestman, W. R., & van den Berg, A. V. (2005). Distributions of alternation rates in various forms of bistable perception. *Journal of Vision*, *5*, 287–298.

Bruza, P., & Cole, R. (2005). Quantum logic of semantic space: An exploratory investigation of context effects in practical reasoning. In S. Artemov, H. Barringer, A. S. dAvila Garcez, L. C. Lamb, & J. Woods (Eds.), *We will show them: Essays in honour of Dov Gabbay. Vol. 1* (pp. 339–361). London: College Publications.

Busemeyer, J., Wang, Z., & Townsend, J. T. (2006). Quantum dynamics of human decision making. *Journal of Mathematical Psychology*, *50*, 220–241.

d'Espagnat, B. (1979). The quantum theory and reality. *Scientific American*, *241*, 158–181.

deWitt, B. S. (1970). Quantum mechanics and reality. *Physics Today*, (Sept.), 30–35.

Einstein, A., Podolsky, B., & Rosen, N. (1935). Can quantum-mechanical description of physical reality be considered complete? *Physical Review*, *47*, 777–780.

Eisert, J., Wilkens, M., & Lewenstein, M. (1999). Quantum games and quantum strategies. *Physical Review Letters*, *83*, 3077–3080. Comment by S.C. Benjamin and P.M. Hayden, Physical Review Letters 87, 069801 (2001), and reply by the authors, Physical Review Letters 87, 069802 (2001).

Everett, H. (1957). Relative state formulation of quantum mechanics. *Reviews of Modern Physics*, *29*, 454–462.

Favrholdt, D. (Ed.), (1999). *Niels Bohr collected works. Volume 10. Complementarity beyond physics* (pp. 1928–1962). Elsevier: Amsterdam.

Filk, T., & von Müller, A. (2009). Quantum physics and consciousness: The quest for a common conceptual foundation. *Mind and Matter*, *7*, 59–79.

Franck, G., & Atmanspacher, H. (2008). A proposed relation between intensity of presence and duration of nowness. In H. Atmanspacher, & H. Primas (Eds.), *Recasting reality – Wolfgang Pauli's philosophical ideas in contemporary science* (pp. 211–225). Berlin: Springer.

Gustafson, K., & Misra, B. (1976). Canonical commutation relations of quantum mechanics and stochastic regularity. *Letters in Mathematical Physics*, *1*, 275–280.

Holton, G. (1970). The roots of complementarity. *Daedalus*, *99*, 1015–1055.

Kochen, S., & Specker, E. (1967). The problem of hidden variables in quantum mechanics. *Journal of Mathematics and Mechanics*, *17*, 59–87.

Kornmeier, J., Ehm, W., Bigalke, H., & Bach, M. (2007). Discontinuous presentation of ambiguous figures: How interstimulus-interval duration affects reversal dynamics and ERPs. *Psychophysiology*, *44*, 552–560.

Leggett, A. J., & Garg, A. (1985). Quantum mechanics versus macroscopic realism: Is the flux there when nobody looks? *Physical Review Letters*, *54*, 857–860.

Long, G. M., & Toppino, T. C. (2004). Enduring interest in perceptual ambiguity: Alternating views of reversible figures. *Psychological Bulletin*, *130*, 748–768.

Mahler, G. (1994). Temporal Bell inequalities: A journey to the limits of consistent histories". In H. Atmanspacher, & G. Dalenoort (Eds.), *Inside versus outside. Endo- and exo-concepts of observation and knowledge in physics, philosophy and cognitive science* (pp. 195–205). Berlin: Springer.

Mahler, G. (1997). Nonlocality in quantum dynamics. In H. Atmanspacher, & E. Ruhnau (Eds.), *Time, temporality, now* (pp. 347–363). Berlin: Springer.

Mermin, N. D. (1990). What's wrong with these elements of reality? *Physics Today*, (June), 9; See also: Quantum mysteries revisited, *American Journal of Physics, 58*, 731–734 (1990).

Misra, B. (1978). Nonequilibrium entropy, Lyapounov variables, and ergodic properties of classical systems. *Proceedings of the National Academy of Sciences of the United States of America*, *75*, 1627–1631.

Misra, B., & Sudarshan, E. C. G. (1977). The Zeno's paradox in quantum theory. *Journal of Mathematical Physics*, *18*, 756–763.

Necker, L. A. (1832). Observations on some remarkable phenomenon which occurs in viewing a figure of a crystal or geometrical solid. *The London and Edinburgh Philosophy Magazine and Journal of Science*, *3*, 329–337.

Orbach, J., Zucker, E., & Olson, R. (1966). Reversibility of the Necker cube: VII. Reversal rate as a function of figure-on and figure-off durations. *Perceptual and Motor Skills*, *22*, 615–618.

Pöppel, E. (1997). A hierarchical model of temporal perception. *Trends in Cognitive Science*, *1*, 56–61.

Primas, H. (2007). Non-Boolean descriptions for mind-matter problems. *Mind and Matter*, *5*, 7–44.

Primas, H. (2008). Complementarity of mind and matter. In H. Atmanspacher, & H. Primas (Eds.), *Recasting reality – Wolfgang Pauli's philosophical ideas in contemporary science* (pp. 171–209). Berlin: Springer.

Sudarshan, E. C. G. (1983). Perception of quantum systems. In A. van der Merwe (Ed.)., *Old and new questions in physics, cosmology, philosophy, and theoretical biology* (pp. 457–467). New York: Plenum.

Tjøstheim, D. (1976). A commutation relation for wide sense stationary processes. *SIAM Journal on Applied Mathematics*, *30*, 115–122.

vanRullen, R., Reddy, L., & Koch, C. (2005). The continuous wagon wheel illusion is associated with changes in electroencephalogram power at ≈13 Hz. *Journal of Neuroscience*, *26*, 502–507.

vanRullen, R. (2009). Private communication.